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Université A-Mira de Bejaia
Faculté des Sciences Exactes
Département de Mathématiques
Laboratoire de Mathématiques Appliquées

University A. Mira of Béjaia (Algeria)

International Conference in Algebra, Geometry and Applications (ICAGA 26)

May 17–20, 2026.

Contents

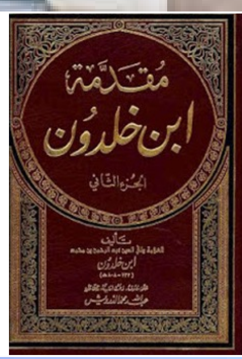
1 University Abderahmane Mira, Béjaia

Founded in 1983 with 205 students and 40 teachers, the University of Béjaïa is among the fastest-growing universities in the world. Currently, it has more than 45 700 students, 1714 teachers-researchers and 1227 administrative and technical staff. The university includes 8 faculties: Faculty of Technology - Faculty of Exact Sciences - Faculty of law and Political Sciences - Natural and Life Sciences- Faculty of Literature and Languages- Human and Social Sciences- Economic Sciences, Management and Commercial Sciences- Medical Sciences. Moreover, the university of Béjaia implements programs more and more aligned with labor market. This approach enabled it to identify the needs of its partners of economic sector in term of human resources and skills.

University of Béjaia has, currently, about thirty research laboratories, approved by the Ministry of Higher Education and Scientific Research dealing with several fields: Modeling and optimization of systems- Materials and process engineering Technology - Organic materials - Environmental engineering - Hydraulics - Information and industrial technology - Biomathematics, Biophysics Biochemistry - Mathematics and Applied Mathematics - Theoretical physics - Ecology and environment - Economy and development - Applied microbiology - Applied biochemistry - Training in applied languages and engineering languages in multilingual environment - aquaculture and marine ecosystems. University of Béjaia implements several development research projects. In particular: technological incubator - Center of innovation and technology transfer - national center of research in agri-food technology, in order to make research in the level of national and international competition and provide a positive dynamics to its growth and development. Highly open on its socio-economic sector, university of Béjaia has been constantly working to encourage sustainable development and to be in harmony with challenges of globalization. The connection between university and local economic sector is an important objective, a challenge and a strategy of Béjaia university in order to implement promising projects. Therefore, a lot of cooperation agreements were signed with national companies. The University/company partnership becomes one of the priority themes in a changing economic environment. In this context, and since 2007, a forum on university and productive world is organized each academic year. It is considered as a space of exchanges and debates on current scientific and socio-economic themes. Therefore, the university has implemented an office of connection between the university and companies (BLEU).

The University of Béjaia has always been committed to a dynamic of intense international scientific cooperation. The desire of university to open to the world is reflected by signing more than sixty cooperation agreements with universities of many countries (France, Italy, Russia, Spain, Romania, Canada, Ukraine, Tunisia ... etc). These agreements were established in order to facilitate scientific exchanges and mobility of teachers researchers, students and administrative staff. The university aims to build a bridge of experience and competences exchange and also improvement of pedagogical methods, enhance the scientific research and establish a strong and sustainable cooperation network. The University of Béjaia is a member of various international academic exchange networks: Averoès Erasmus Mundus, TEMPUS, CMEP, DEF/CNRS, and PCIM.

Website: https://www.univ-Béjaia.dz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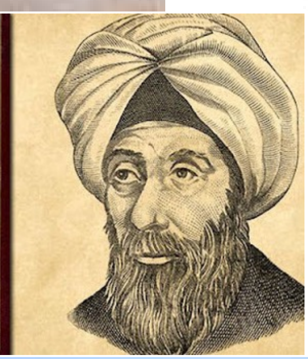

2 Béjaia city

Béjaia (Bgayet in berber) is a beautiful city in the Mediterranean, located between the majestic Djurdjura, Bibans, and Babors mountain ranges. Béjaia is one of Algeria's oldest cities, founded in 26-27 BC by Emperor Augustus under the name Saldae. Béjaia played a significant role in commercial activity across the Mediterranean as Phoenician trading post. During the Middle Ages, the city became one of the most prosperous in the Mediterranean, as well as a major intellectual center. In 1067, Prince El Hammad Ennacer founded a new city there, which he named En Naciria. It later became the capital of the Hammadid kingdom after the conflicts between the Almoravids and the Zirids. However, in 1152, it experienced the rise of the Almohads, who transformed it into a province. The city of learning played a prominent role, attracting prestigious historical, scientific, and literary figures from all fields. The Sicilian poet Ibn Hamadi, the Italian mathematician Leonardo Fibonacci, the Catalan philosopher Raymond Lull, the historian Ibn Khaldun, the Andalusian metaphysician Ibn Arabi, as well as highly regarded religious figures such as Sidi Boumediène and Athaalibi, resided in Béjaia for a time. Furthermore, Mahdi Ibn Toumert carried out his reformist activities there, meeting Abdel Moumène in Mellala, a small village near Béjaia. Scholars came to complete their training in the city as they did in Cairo, Tunis, Fes or Tlemcen. Hundreds of students, some of European origin, crowded into the schools and mosques where theologians, jurists, philosophers and scholars taught. The main places of medieval knowledge were the Great Mosque, Madinat al-Ilm (the City of Sciences), the Khizana Sultaniya and the Sidi Touati Institute.

In 1202, Leonardo Fibonacci, an Italian mathematician, brought back the ”Arabic numerals” and the algebraic notation. According to the versions, the inspiration for the Fibonacci sequence would be due to the observation of beekeepers and the reproduction of bees in the region or to a local mathematical problem concerning the reproduction of rabbits that he describes in his work Liber abaci.

In 1510, Béjaia fell to the Spanish, who destroyed more than half the city. In 1555, Salah Raïs drove the Spanish out of the city but failed to restore its former glory. In 1833, the French took Béjaia and subsequently endowed it with numerous amenities, including boulevards, villas, apartment buildings, and barracks, which still bear witness to this long history under various occupations.

Béjaia remains a city rich in an immeasurable cultural treasure, a treasure of monuments and historical sites which are the subject of a vast rehabilitation operation to restore each monument to its value and jealously guard this heritage for future generations. The citadel, the city's most important historical monument, is the product of the interaction of various cultures: Roman, Hammadid, Spanish, Turkish, French, and Arab-Muslim.

Currently, Béjaia is the largest city in Kabylia region with about 180 000 inhabitants. Thanks to its geographical location and dynamism, it is also the most important industrial center in the region, notably due to the concentration of numerous industries and the presence of one of the largest oil and commercial harbors in the Mediterranean. Its tourism potential also places it as one of the most popular destinations in Algeria for national and international tourists.

3 Conference Introduction

The aim of the conference is to bring together experts and young researchers from all over the world working on Algebra, Geometry and various related fields. The objective is to provide a comprehensive overview of recent developments in nonassociative algebras, Lie theory, Hopf algebras, differential geometry and mathematical physics. The event will foster a collaborative and supportive research environment offering an opportunity, in particular for Algerian young researchers, to exchange ideas and insights, and build relationships with recognized personalities attending the conference.

The topics covered are:

- Nonassociative algebras and Lie Theory
- Quantum groups and Hopf algebras
- Differential Geometry
- Applications: Mathematical Physics

4 Scientific Committee

- **Efim Zelmanov** (Shenzhen International Center of Mathematics, China)
- **Lei Ni** (Zhejiang Normal University, China)
- **Vyacheslav Futorny** (Shenzhen International Center of Mathematics, China)
- **Chengming Bai** (Chern Institute, Nankai University, China)
- **Boujemaa Agrebaoui** (Sfax University, Tunisia)
- **Ali Baklouti** (Sfax University, Tunisia)
- **Martin Bordemann** (University of Haute Alsace, France)
- **Abdenacer Makhlouf** (University of Haute Alsace, France)
- **Sofiane Bouarroudj** (New York University, Abu Dhabi, UAE)
- **Claudia Menini** (Ferrara University, Italy)
- **Blas Torrecillas** (Almeria University, Spain)
- **Tahar Boujedaa** (Jijel University)

5 Organizing Committee

- Abdelkrim Beniaiche (Rector of Béjaia University)
- Mounya Belhocine (Vice-Rector in charge of International relations)
- Sofiane Aoudia (Dean of Exact Science Faculty)
- Said Aissaoui (Conference Chair)
- Abdenacer Makhlouf (Conference coChair)
- Arezki Kheloufi (Conference coChair)
- Karima Mebarki
- Hakim Moussaoui
- Louiza Berdjoudj
- Mohand Bouraine
- Mohand Said Boukhelifa
- Boudjamaa Kerai
- Karima Timeridjine
- Samira Zahar
- Rachid Boukoucha
- Mohamed Mouzaia
- Fouad Maouche
- Nadir Salhi
- Yacine Boumzaid
- Ouazine Sofiane
- Nour El Houda Kadi
- Yaniss Yahiaoui
- Khaled Kessoum
- Toufik Mekerri

6 Invited Speakers

Efim Zelmanov (Shenzhen Intenational Center of Mathematics, China)
Abdenacer Makhlouf (Université de Haute Alsace, France)
Vyacheslav Futorny (Shenzhen Intenational Center of Mathematics, China)
Boujemaa Agrebaoui (Sfax University, Tunisia)
Ali Baklouti (Sfax University, Tunisia)
Abdelghani Zeghib (ENS Lyon, France)
Salah Mehdi (Lorraine University, France)
Sami Mabrouk (Gafsa University, Tunisia)
Hacène Belbachir (USTHB Algeria)
Khireddine Nouicer (Jijel University, Algeria)
Imed Basdouri (Gafsa University, Tunisia)
Taoufik Chtioui (Gabes University, Tunisia)
Sofiane Bouarroudj (New York University, Abu Dhabi, UAE)
Jiping Zhang (Pekin University, China)
Blas Torrecillas (University of Almeria, Spain)
Zoheir Chebel (University Center Barika, Algeria)
Amine Bahayou (Ouargla University, Algeria)
Hadjer Adimi (ENS Sétif, Algeria)
Ahmed Zeglaoui(National Higher School of Autonomous Systems Technology Sidi Abdellah, Algeria)
Mouloud Goubi (Mouloud mamari university, Algeria)
Nizar Ben Fraj (Carthage University, Tunisia)
Xabier Garcia-Martinez (University of Santiago de Compostela, Spain)
Hakim Moussaoui (Ecole ESTIN Béjaia, Algeria)
Salaheddine Rihane (National Higher School of Mathematics, Algeria)
Nadir Trabelsi (University of Sétif, Algeria)
Salah Haouat (Jijel University, Algeria)
Noureddine Ferkous (Jijel University, Algeria)
Abdennour Kitouni (Constantine University, Algeria)

7 List of participants

Boujemaa Agrebaoui (Sfax University, Tunisia)
Ali Baklouti (Sfax University, Tunisia)
Abdelghani Zeghib (ENS Lyon, France)
Salah Mehdi (Lorraine University, France)
Sami Mabrouk (Gafsa University, Tunisia)
Abdenacer Makhlouf (Université de Haute Alsace, France)
Hacène Belbachir (USTHB Algeria)
Khireddine Nouicer (Jijel University, Algeria)
Imed Basdouri (Gafsa University, Tunisia)
Quentin Ehret (New York University, Abu Dhabi, UAE)
Taoufik Chtioui (Gabes University, Tunisia)
Sofiane Bouarroudj (New York University, Abu Dhabi, UAE)
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Okbani Hadjer (University of Mascara, Algeria)
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Benkartab Aicha (University of Mascara, Algeria)
Youssra Miloudi (ENS Kouba, Algeria)
Foukroun Nadjiba. (USTHB, Algeria)
Amel Dilmi (University of Setif 1, Algeria)
Amel Bouraada (University of Mascara, Algeria)
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Fedala Dyhia (USTHB, Algeria)
Chemikh Smail (USTHB, Algeria)
Tadj Moustafa. (University of Naama , Algeria)
Zegga Kaddour (University of Mascara, Algeria)
Boucharef Kaouther (University of Bordj Bou Arreridj , Algeria)
Merdji Bouchra (University of Mascara, Algeria)
Boukhari Halima (University of Mascara, Algeria)
Benallou Mohamed (University of Tiaret, Algeria)
Nassamou Touhami (University of Naama, Algeria)
Sidhoumi Noura (Ecole Nationale Polytechnique of Oran, Algeria)
Seddik Ouakkas (ENS Saida, Algeria)
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Fernane. Zahia (USTHB, Algeria)
Kheloufi Naima (USTHB, Algeria)
Abdelbasset Hasni (University of Mascara , Algeria)
Benabdallah Seyyid Ali (USTHB, Algeria).
Abbad Sadjia (University of Blida 1, Algeria).
Zagane Abdelkader (University of Mascara , Algeria)
Tahri Kamel (University of Tlemcen, Algeria)
Hanene Amri (University of Annaba)
Said Aissaoui (University of Béjaia)
Arezki Kheloufi (University of Béjaia)
Karima Mebarki (University of Béjaia)
Louiza Berdjoudj (University of Béjaia)

Mohand Bouraine (University of Béjaia)
Mohand Said Boukhelifa (University of Béjaia)
Boudjamaa Kerai (University of Béjaia)
Karima Timeridjine (University of Béjaia)
Samira Zahar (University of Béjaia)
Rachid Boukoucha (University of Béjaia)
Mohamed Mouzaia (University of Béjaia)
Fouad Maouche (University of Béjaia)
Nadir Salhi (University of Béjaia)
Yacine Boumzaid (University of Béjaia)
Ouazine Sofiane (University of Béjaia)
Nour El Houda Kadi (University of Béjaia)
Yaniss Yahiaoui (University of Béjaia)
Khaled Kessoum (University of Béjaia)
Toufik Mekerri (University of Béjaia)
Mourad Mehidi (University of Béjaia)

8 Programme

Sunday, May 17th, 2026

9h00-9h25 Opening
9h25-10h10 Hacène Belbachir, USTHB Algiers

10h10 Coffee break

10h45-11h30 Ali Baklouti, Sfax University, Tunisia
11h30-12h15 Abdelghani Zeghib, ENS Lyon, France

12h10 Lunch

14h00-14h45 Sofiane Bouarroudj, New York University, Campus Abu Dhabi
14h45-15h30 Nadir Trabelsi, Sétif University

15h30 Coffee break

16h00 -16h35 Sami Mabrouk, Gafsa University, Tunisia
16h35 -17h10 Mouloud Goubi, Tizi Ouzou University
17h10 -17h45 Salah Haouat, Jijel University
17h45 -18h15 Amine Bahayou, University of Ouargla

Monday, May 18th, 2026

8h30-9h15 Khireddine Nouicer, Jijel University
9h15-10h00 Blas Torrecillas, Almeria University, Spain

10h00 Coffee break

10h30-11h15 Salah Mehdi, Lorraine University, France
11h15-11h50 Taoufik Chtioui, Gabes University, Tunisia
11h50-12h20 Hakim Moussaoui, Higher School of Computer Science and Digital Technologies of Bejaia

13h00 Lunch

14h30 Excursion

Tuesday, May 19th, 2026

8h30-9h15 Boujemaa Agrebaoui, Sfax University, Tunisia
9h15-10h00 Ahmed Zeglaoui, National Higher School of Autonomous Systems Technology (NHSAST)

10h00 Coffee break

10h30-11h15 Xabier Garcia-Martinez, University of Santiago, Spain
11h15-12h00 Imed Basdouri, Gafsa University, Tunisia
12h00-12h30 Nourredine Ferkous, Jijel University

12h30 Lunch

14h00-15h30 Selected Communications and parallel sessions

15h30 Coffee break

16h00-18h30 Selected Communications and parallel sessions

Wednesday, May 20th, 2026

8h30-9h15 Abdenacer Makhlouf, University of Haute Alsace, France
9h15-10h00 Nizar Ben Fraj, Carthage University, Tunisia

10h00 Coffee break

10h30-11h05 Salaheddine Rihane, National Higher School of Mathematics
11h05-11h40 Zoheir Chebel, Barika University Center
11h40-12h15 Abdennour Kitouni, Constantine University

12h15 Closing

9 Talks : Titles and Abstracts

Enumerating step-constrained self-avoiding walks on lattices

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Abstract.

The study of self-avoiding walks (SAWs) on integer lattices has been an area of active research for several decades. Our aim is to investigate the number of SAWs between two diagonally opposite corners in a finite rectangular subgraph of the integer lattice, subject to certain constraints. In the two-dimensional case, an explicit formula for the number of SAWs of a prescribed length is presented, restricted to three-step directions: up (↑), left (←) considered as the wrong step, and right (→).

References

[1] N. R Beaton and A. L Owczarek. Exact solution of weighted partially directed walks crossing a square. J. Phys. A: Math. Theor., 56(15):155003, 2023.

[2] H. Belbachir, L. Major, L. Németh, and L. Szalay. Step-constrained self-avoiding walks on finite grids. J. Comb. Theory, Ser. A 218, Article ID 106104, 23 p. 2026.

[3] M. Bousquet-Mélou, A. J. Guttmann, and I. Jensen. Self-avoiding walks crossing a square. J. Phys. A: Math. Gen., 38(42):9159, 2005.

[4] T. W. Burkhardt and I. Guim. Self-avoiding walks that cross a square. J. Phys. A: Math. Gen., 24(20): L1221, 1991.

[5] A. J. Guttmann, I. Jensen, and A. L. Owczarek. Self-avoiding walks contained within a square. J. Phys. A: Math. Theor., 55(42):425201, oct 2022.

[6] M. Liskiewicz, M. Ogihara, and S. Toda. The complexity of counting self-avoiding walks in subgraphs of two-dimensional grids and hypercubes. Theoret. Comput. Sci., 304(1-3):129–156, 2003.

[7] N. Madras. Critical behaviour of self-avoiding walks: that cross a square. J. Phys. A Math. Theor., 28(6):1535, 1995.

[8] N. Madras and G. Slade. The self-avoiding walk. Springer Science and Business Media, 2013.

Groups in which all proper subgroups satisfy certain properties

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Abstract.

Let Ω be a class of groups. A group G is called a minimal non-Ω group, a MNΩ group in short, if all its proper subgroups belong to Ω, while G itself is not in Ω. The study of MNΩ groups started in the beginning of the last century when Miller and Moreno in 1903 [8] and Schmidt in 1924 [10], characterized finite MN$\mathfrak{A}$ groups and finite MN$\mathfrak{N}$ groups respectively, where $\mathfrak{A}$ and $\mathfrak{N}$ denote the classes of abelian and nilpotent groups respectively. Among the first papers that deal with infinite MNΩ groups, one can cite the paper of Newman and Wiegold in 1964 [9], in which there is a complete description of locally nilpotent MN$\mathfrak{N}$ groups that have maximal subgroups. Locally nilpotent MN$\mathfrak{N}$ groups without maximal subgroups have been studied by Smith in 1997. Since the paper [9] was published, many authors studied the structure of infinite MNΩ groups for different choice of Ω, in particular, when Ω is a subclass of the class of (locally nilpotent)-by-(locally finite) or (locally finite)-by-(locally nilpotent) groups (see for instance [1], [2], [6]. Recall that a locally graded group is a group in which every non-trivial finitely generated subgroup has a non-trivial finite homomorphic image. The study of infinite MNΩ groups is done within the universe of locally graded groups in order to avoid Tarski groups, that is infinite finitely generated simple groups in which every proper subgroup is of prime order, hence these groups are MNΩ groups for almost all classes Ω of groups, but they have a complicated structure. Asar, in 2000 [1], proved that a locally graded group whose proper subgroups are nilpotent-by-Chernikov is itself nilpotent-by-Chernikov. Badis and Trabelsi in 2011 [2], have obtained a similar result for the class of Baer-by-Chernikov groups, by proving that a locally graded group whose proper subgroups are Baer-by-Chernikov is itself Baer-by-Chernikov. Recall that a Baer group is a group in which each cyclic subgroup is subnormal. In this work, we will add a similar result for the class of (locally nilpotent)-by-Chernikov groups. More precisely, we proved the following result.

Theorem 9.1. *A locally graded group whose proper subgroups are (locally nilpotent)-by-Chernikov is itself (locally nilpotent)-by-Chernikov.*

A group G is said to be of finite rank a positive integer r, if every finitely generated subgroup of G can be generated by at most r elements, and r is the least positive integer with such property. If there is no such r, then the group G is said to be of infinite rank.

In recent years, many authors studied the structure of locally (soluble-by-finite) groups G of infinite rank in which every subgroup of infinite rank belongs to a given class Ω and they proved that all proper subgroups of G belong to Ω, sometimes that the group G itself belongs to Ω (see for instance, [3],[4],[5],[7]).

In the present talk, we will consider this problem for the class of (locally nilpotent)-by-Chernikov groups. More precisely, we proved the following result.

Theorem 9.2. *If G is a locally (soluble-by-finite) group of infinite rank whose proper subgroups of infinite rank are (locally nilpotent)-by-Chernikov, then it is (locally nilpotent)-by-Chernikov.*

The reason of the restriction to the class of locally (soluble-by-finite) groups is that the behavior of locally graded groups of finite rank is unknown.

References

[1] A. O. Asar, *Locally nilpotent p-groups whose proper subgroups are hypercentral or nilpotent-by-Chernikov*, J. London Mat. Soc. (2) **61** (2000), 412-422.

[2] A. Badis and N. Trabelsi, *Groups whose proper subgroups are Baer-by-Chernikov*, Cent. Eur. J. Math. **9** (2011), 1344-1348.

[3] M. De Falco, F. de Giovanni, C. Musella and N. Trabelsi, *Groups with restrictions on subgroups of infinite rank*, Rev. Mat. Iberoamericana **30** (2014), 537-550.

[4] A. Dilmi and N. Trabelsi, *Groups whose proper subgroups of infinite rank are finite-by-hypercentral or hypercentral-by-finite*, Acta Math. Hung. **167** (2022), 492-500.

[5] M. R. Dixon, M. J. Evans and H. Smith, *Locally (soluble-by-finite) groups with all proper non-nilpotent subgroups of finite rank*, J. Pure Appl. Algebra **135** (1999), 33-43.

[6] M. R. Dixon, M. J. Evans and H. Smith, *Groups with all proper subgroups nilpotent-by-finite rank*, Arch. Math. **75** (2000), 81-91.

[7] F. de Giovanni and M. Trombetti, *Groups of infinite rank with a locally finite term in the lower central series*, Beitr. Algebra Geom. **56** (2015), 735-741.

[8] G. A. Miller and H. C. Moreno, *Non-abelian groups in which every subgroup is abelian*, Trans. Amer. Math. Soc. **4** (1903), 398-404.

[9] M. F. Newman and J. Wiegold, *Groups with many nilpotent subgroups*, Arch. Math. **15** (1964), 241-250.

[10] O. J. Schmidt, *Uber Gruppen, deren samtliche Teiler spezielle Gruppen sind*, Rec. Math. Moscow **31** (1924), 366-372.

About a Conjecture of Michel Duflo

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Abstract

Let $G = \exp \mathfrak{g}$ be a connected and simply connected real nilpotent Lie group with Lie algebra $\mathfrak{g}$, $H = \exp \mathfrak{h}$ an analytic subgroup of G with Lie algebra $\mathfrak{h}$, χ a unitary character of H and $\tau = \mathrm{ind}_H^G \chi$ the monomial representation of G induced from χ. Let $D_\tau(G/H)$ be the algebra of the G-invariant differential operators on the line bundle over G/H associated to the data (H, χ). We know that χ is written as χ_f, where $\chi_f(\exp X) = e^{if(X)}(X \in \mathfrak{h})$ with a certain $f \in \mathfrak{g}^*$ verifying $f([\mathfrak{h}, \mathfrak{h}]) = \{0\}$. Let $S(\mathfrak{g})$ be the symmetric algebra of $\mathfrak{g}$ and $\mathfrak{a}_\tau = \{X + \sqrt{-1} f(X); X \in \mathfrak{h}\}$. Then, $S(\mathfrak{g})$ possesses the Poisson structure $\{,\}$ well determined by the equality $\{X, Y\} = [X, Y]$ if X, Y are in $\mathfrak{g}$. We consider the algebra $(S(\mathfrak{g})/S(\mathfrak{g})\overline{\mathfrak{a}_\tau})^H$ of H-invariant polynomial functions on the affine subspace $\Gamma_\tau = \{\ell \in \mathfrak{g}^* : \ell(X) = f(X), X \in \mathfrak{h}\}$ of $\mathfrak{g}^*$. This inherits the Poisson structure from $S(\mathfrak{g})$. We denote by Z_τ its Poisson center and C_τ the center of $D_\tau(G/H)$. Duflo's polynomial conjecture (cf. [1]) states that Z_τ and C_τ are isomorphic as algebras. The talk plans to explain some steps toward the proof, generalizing some particular cases (cf. [2]).

This is a joint work with H. Fujiwara.

References

[1] M. Duflo, "Open problems in representation theory of Lie groups" in: Conference on Analysis on Homogeneous Spaces, T. Oshima (ed.), August 25-30, Katata in Japan 1986, p. 1-5.

[2] Y. Tanimura, " An algebraic approach to Duflo's polynomial conjecture in the nilpotent case", *J. Lie Theory*, vol. 29, 2019, p. 839-879.

Bieberbach rigidity of locally homogeneous spaces

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Abstract. The talk surveys the notion of Bieberbach rigidity for local models of homogeneous spaces, defined as follows. Let $X = G/H$ be a homogeneous space, and consider a discrete subgroup $\Gamma \subset G$ acting (on the left) properly and cocompactly on X. This means that the quotient $M = \Gamma\backslash X$ is a compact Hausdorff space. The situation is well understood when X is of Riemannian type, meaning that the isotropy subgroup H is compact. Equivalently, in this case, the full group G acts properly on X. Then, the condition on Γ reduces to requiring that Γ is a cocompact lattice in G. Our focus, however, is on non-Riemannian homogeneous spaces, i.e., those with non-compact isotropy H. In this setting, Γ cannot be a cocompact lattice in G; otherwise, G itself would act properly on X, which would contradict the fact that X is non-Riemannian. Thus, Γ must be thinner than a lattice, yet large enough to produce a compact quotient. A particularly interesting situation arises when (up to finite index) every such discrete group Γ is contained in a connected closed subgroup $L \subset G$ acting properly on X, so that Γ is in fact a lattice in L. One may view L as a connected hull of Γ, reminiscent in many cases of its Zariski closure. The guiding philosophy is that studying connected Lie subgroups $L \subset G$ acting properly and cocompactly - a problem of linear-algebraic nature - is generally far more tractable than analyzing discrete subgroups Γ directly - which has a number-theoretic flavor. The homogeneous space X is said to satisfy Bieberbach rigidity if all its compact quotients M arise in this way.

Flat pseudo-Euclidean Leibniz superalgebras

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Abstract.

Wee introduce pre-Lie and pre-Leibniz superalgebras, which generalize pre-Lie and pre-Leibniz algebras to the super setting. Additionally, we define a Levi-Civita product associated with a non-degenerate symmetric bilinear form on a non-associative superalgebra. This leads to the definition of flat pseudo-Euclidean left Leibniz superalgebras as those whose Levi-Civita product induces a pre-Leibniz structure. We show that a quadratic Leibniz superalgebra is flat if and only if it is symmetric Leibniz and 2-step nilpotent. Finally, we introduce the notion of double extension for flat pseudo-Euclidean (resp. Lie) left Leibniz superalgebras and prove that any flat pseudo-Euclidean non-Lie left Leibniz superalgebra can be obtained by a sequence of double extensions starting from a flat pseudo-Euclidean Lie superalgebra.

This is a joint work with S. Benayadi and H. El Ouali.

Hessian- Poisson manifolds

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Abstract.

A Hessian structure is defined by a Riemannian metric on a locally flat manifold satisfying the Codazzi equation. An example of a Hessian structure is that defined on a domain of $\mathbb{R}^n$ using the Fisher metric associated with an exponential statistical model.
The Riemannian metric underlying a Hessian structure allows us to define the dual Hessian structure in the sense of Amari-Chentsov.

A Hessian-Poisson structure (or Hessian-Poisson manifold) and its dual structure are defined by analogy to the case of a Hessian structure ; the Riemann-Poisson manifold replaces the underlying Riemannian manifold.

References

[1] S. I. Amari, H. Nagaouka, *Methods of Information Geometry*, Translations of Mathematical Monographs, vol. 191.

[2] M. Boucetta, *Compatibilité des structures pseudo-riemanniennes et des structures de Poisson*, C. R. Acad. Sci. Paris, Ser. I 333 (2001) 763-768.

[3] M. Boucetta, *Poisson manifolds with compatible pseudo-metric and pseudo-Riemannian Lie algebras*, Differential Geometry and its applications, 20 (2004), 279-291.

[4] J.-P. Dufour, N. T. Zung, *Poisson structures and their normal forms*, Pogress in mathematics, vol. 242., Birkhäuser Verlag 2005.

[5] R. L. Fernandes, *Lie algebroids, holonomy and characteristic classes*, Adv. Math., 170, (2002) 119-179.

[6] M. Nguiffo Boyom, *The cohomologie of Koszul-Vinberg algebras*, Pacific Journal of Mathematics, vol. 225, N° 1, 2006, pp 119-153. DOI: 10.2140/pjm.2006.225.119.

[7] M. Nguiffo Boyom, *Foliations-Webs-Hessian Geometry-Information Geometry and Cohomology*, Entropy 2016, 18, 433.

[8] H. Shima, *The geometry of hessian structures*, World Scientific Singapore Co. Pte. Ltd., NJ 07601, 2007.

[9] I. Vaisman, *Lectures on the geometry of Poisson manifolds*, Progress in Mathematics, vol 118, Birkhäuser, Berlin, 1994.

The invariant ring of pairs of matrices

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Abstract.

Let us consider the action of the general linear group $\mathrm{GL}_n(\mathbb{C})$ on the direct product $\mathcal{M}_n^d$ of d copies of $\mathcal{M}_n$ by simultaneous conjugation sending $(X_1, ..., X_d)$ to $(gX_1g^{-1}, \ldots, gX_dg^{-1})$ for any $g \in \mathrm{GL}_n(\mathbb{C})$.

This induces an action of $\mathrm{GL}_n(\mathbb{C})$ on the algebra $\mathbb{C}[\mathcal{M}_n^d]$ of polynomial functions on $\mathcal{M}_n^d$.

The algebra of invariants under this action, $\mathbb{C}[\mathcal{M}_n^d]^{\mathrm{GL}_n}$, is an important object in several areas of mathematics.

In this talk we will explain how we used methods coming from non-associative algebras to obtain the full description of the case $\mathbb{C}[\mathcal{M}_4^2]^{\mathrm{GL}_4}$, which could not be solved using the standard representation theory methods. Moreover, we will talk about its connection with the Calogero-Moser spaces and the Hilbert scheme of points.

References

[1] F. Eshmatov, X. García-Martínez and R. Turdibaev.

Noncommutative Poisson structure and invariants of matrices

Advances in Mathematics, 469, 110212, (2025).

[2] F. Eshmatov, X. García-Martínez, T. Normatov and R. Turdibaev.

On the coordinate rings of Calogero-Moser spaces and the invariant commuting variety of a pair of matrices.

Results in Mathematics, 80, Article 68, (2025).

[3] X. García-Martínez, T. Normatov and R. Turdibaev.

The ring of invariants of pairs of 3×3 matrices.

Journal of Algebra, 603(1), 201–212, (2022).

Conformal Killing Gravity in (1+3) and (1+2) Dimensions

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Joint work with Gérard CLÉMENT
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Abstract.

This talk presents the recently proposed theory of gravity by Junpei Harada (2023), known as Conformal Killing Gravity, a third-order extension of general relativity. In (1+3)-dimensional spherically symmetric spacetimes with Maxwell fields, the theory admits solutions characterized by six independent parameters, including non asymptotically flat black holes, naked singularities, traversable wormholes, and closed (potentially singularity-free) universes. We also investigate sourceless, time-dependent solutions in FRW spacetimes. These exhibit a rich range of cosmological behaviors, from singularity-free eternal evolutions to symmetric universes evolving from a big bang to a big crunch in finite time. In (1+2) dimensions, we focus on the sourceless case, which yields black hole and wormhole solutions. We conclude with a brief discussion of the theory's limitations and open challenges.

References

[1] J. Harada, "Gravity at cosmological distances: Explaining the acceler- ating expansion without dark energy", Phys. Rev. D 108 (2023) 044031 [arXiv:2308.02115].

[2] C.A. Mantica and L.G. Molinari, "Note on Harada's conformal Killing gravity", Phys. Rev. D 108 (2023) 124029 [arXiv:2308.06803].

[3] Gérard Clément and Khireddine Nouicer, Spherical symmetric solutions of conformal Killing gravity: black holes, wormholes, and sourceless cosmologies, Classical Quantum Gravity 41, 165005 (2024).

[4] A. Barnes, "Spherically symmetric electrovac spacetimes in conformal Killing gravity", Class. Quantum Grav. 41, 155007 (2024).

[5] Harada, J., Emergence of the Cotton tensor for describing gravity, Phys. Rev. D 103, L121502 (2021) [arXiv:2105.09304v2 [gr-qc]]

[6] Harada, J., Cotton gravity and 84 galaxy rotation curves, Phys. Rev. D, 106, 064044 pp., (2022).

[7] Gérard Clément and Khireddine Nouicer, Farewell to Cotton gravity, LAPTH-007/24 [arXiv:2401.16008v2 [gr-qc]]

Double Wreath Quasi-Hopf Algebras

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Abstract.

For a quasi-bialgebra H, we show that the category C of H-bimodules is duoidal and that the so-called u-coaugmented bimonoids in C are exactly the quasi-bialgebras with a coalgebra projection. When H is a quasi-Hopf algebra with bijective antipode, we prove that the u-coaugmented bimonoids in C can also be described as what we will call double wreath quasi-Hopf algebras, objects determined by H and pre-bialgebras R within the category of Yetter-Drinfeld modules over H. A particular class of double wreath quasi-Hopf algebras is obtained by deforming with a 2-cocycle the multiplication of a Radford biproduct quasi-Hopf algebra. Other classes of this type are given by the symplectic fermion quasi-Hopf algebras.

This is a joint work with D. Bulacu and D. Popescu.

From characters to cohomology : An asymptotic Dirac perspective

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Abstract.

Lie groups encode symmetries, and their representations are, in many cases, determined by their characters. Remarkably, Dirac operators, introduced in the late 1920s by Paul Dirac in his description of the quantum dynamics of the electron, provide a powerful and unifying framework for studying representations of Lie groups. In this talk, I will explain how Dirac operators can be used to define and compute important invariants, including sheaves, associated varieties, cycles, nilpotent orbits, and cohomological data, which capture deep structural aspects of representations. If time permits, I will describe recent developments from ongoing joint work with P. Pandžić, D. Vogan, and R. Zierau. Overall, this perspective reveals a lively and rich interplay between quantum physics, algebra, geometry, and analysis.

Solenoidal Lie (super)algebras of contact type, their cuspidal modules and their q-analogues

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Abstract.

$\mathbb{Z}^n$-graded algebras of Witt type and their central extensions are considered by R. Yu in [9]. Recently, Y. Billig and V. Futorny [6], considered the so called solenoidal Lie algebra $\mathcal{W}_\mu$ wich is simple of Witt type and they study its cuspidal modules. For $n = 1$, it is the well known Witt algebra $\mathcal{W}(1) := \mathfrak{Der}(\mathbb{C}[t, t^{-1}])$. In [4], we compute the universal central extension of the solenoidal Lie algebra, we obtain a higher rank Virasoro algebra. For $n = 1$, it is the well known Virasoro algebra-the universal central extension of $W(1)$-generated by the well known Gelfand-Fuks 2-cocycle. Then we study its Harish-Chandra modules. In an other work, [5], we introduce the solenoidal Heisenberg-Virasoro algebras and we study its Harish-Chandra modules.

Lie superalgebras are mathematical structures that extend classical Lie algebras to incorporate $\mathbb{Z}_2$-grading (even and odd parts). The motivation is the need to supersymmetry model in physics (simultaneous description of bosonic (even) and fermionic (odd) degrees of freedom) and advanced algebraic structure in mathematics. Odd elements satisfy anti-commutation relations rather than simple commutation

V. Kac and J. van de Leur [7], give a conjecture on the clasification of superconformal alge bras ($\mathbb{Z}$-graded Lie superalgebras of growth one where the Witt algebra is a subalgebra of the even part). They study their central extensions (again the Virasoro algebra is a subalgebra of the even part). One important class is contact superalgebras. In a recent work (172 pages) C. Martìnez, O. Mathieu and E. Zelmanov [8], describe an explicit classification of all cuspidal modules over the superconformal algebras given in [7] . In a work in progress in collaboration with V. Futorny, we introduce the so called solenoidal contact Lie superalgebras and we compute their central extensions and we study their cuspidal modules.

We will consider also the q-analogues q-$W_\mu(n)$ and q-$HW_\mu(n)$ of $W_\mu(n)$ and $HW_\mu(n)$ respectively. We computed their universal central extensions q-$Vir_\mu(n)$ and q-$HVir_\mu(n)$ and we studied their cuspidal modules [1, 2] in progress.

References

[1] B. Agrebaoui, M. Haddar, A. Hajjaji, W. Mhiri and A. Makhlouf, The q-solenoidal (-Virasoro) algebra and its cuspidal modules.

[2] B. Agrebaoui, M. Haddar and A. Makhlouf, The q-solenoidal (Heisenberg-Virasoro) algebras and its cuspidal modules. Work in progress.

[3] B. Agrebaoui, V. Futorny, The solenoidal contact Lie superalgebras, their central extensions and their cuspidal modules. Work in progress.

[4] B. Agrebaoui, W. Mhiri, The solenoidal Virasoro algebra and its simple weight modules. Communications in algebra,53(4), pp. 1568-1580.

[5] B. Agrebaoui, W. Mhiri, The solenoidal Heisenberg-Virasoro algebra and its simple weight modules. Communications in algebra, to appear.

[6] Y. Billig, V. Futorny, Classification of simple cuspidal modules for solenoidal Lie algebras, Isr.J.Math. 222,(2017), 109-123.

[7] V.G. Kac and J.W. van de Leur, On classification of superconformal algebras. Proceedings of Strings 88 Teaneck (1989) 77-106.

[8] C. Martinez, O. Mathieu and E. Zelmanov, Cuspidal modules over superconformal algebras of rank $\geq$ 1.arXiv:2505.20974 [math.RT]

[9] R. W. T. Yu, Algèbre de Lie de type Witt. Communications in algebra, 25(5), 1471-1484(1997).

q-Deformations, Hom-type and Bihom-type algebraic structures

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Abstract.

A quantum deformation or q-deformation consists of replacing usual derivation by a σ-derivation or (σ, τ)-derivation in algebras of vector fields. The main example is given by Jackson derivative and lead for example to q-deformation of $\mathfrak{sl}_2$, Witt algebra, Virasoro algebra and also Heisenberg algebras (oscillator algebras). The description of the new structures gave rise to a structure generalizing Lie algebras, called Hom-Lie algebras or quasi-Lie algebras studied first by Larsson and Silvestrov. Since then various classical algebraic structures and properties were extended to the Hom-type setting. The main feature is that the classical identities are twisted by homomorphisms.

The purpose of my talk is to give an overview of recent developments and provide some key constructions, examples and relevant results.

References

[1] J. T. Hartwig, D. Larsson, S. D. Silvestrov, *Deformations of Lie algebras using σ-derivations*, J. Algebra **295** (2006), 314–361.

[2] A. Makhlouf, S. Silvestrov, *Hom-algebras and Hom-coalgebras*, J. Algebra Appl. **9** (2010), 553–589.

[3] C. Laurent-Gengoux, A. Makhlouf and J. Teles, *Universal Algebra of a Hom-Lie Algebra and group-like elements*, Journal of Pure and Applied Algebra, **222** no. 5, 1139–1163, (2018)

[4] B. Hurle and A. Makhlouf, *α-type Hochschild cohomology of Hom-associative algebras and bialgebras.* Journal of the Korean Mathematical Society (2019).

[5] G. Graziani, A. Makhlouf, C. Menini, F. Panaite, *BiHom-associative algebras, BiHom-Lie algebras and BiHom-bialgebras*, Symmetry Integrability Geom. Methods Appl. **11** (2015), 086, 34 pages.

[6] A. Gezer, A. Makhlouf, L. Nourmohammadifar and E. Peyghan, *Kähler-Norden structures on Hom-Lie groups and Hom-Lie algebras.* Hacet. J. Math. Stat. Volume 54 (1) (2025), 213 –236

Lie bialgebra structures and the geometry of character varieties for flat metric Lie groups

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Abstract.

Let G be a simply connected Lie group equipped with a left-invariant flat Riemannian metric, and let $\mathfrak{g} = \mathrm{Lie}(G)$. By Milnor's structure theorem [3], the Lie algebra $\mathfrak{g}$ admits an orthogonal decomposition

$$\mathfrak{g} = \mathfrak{s} \oplus \mathfrak{z} \oplus \mathfrak{u},$$

where $\mathfrak{s}$ is an abelian Levi factor, $\mathfrak{z}$ denotes the center, and $\mathfrak{u} = [\mathfrak{g}, \mathfrak{g}]$ is an even-dimensional abelian ideal on which $\mathfrak{s}$ acts by pairwise commuting infinitesimal rotations. This rigid algebraic structure induces a weight-plane decomposition of $\mathfrak{u}$ into real planes P_ℓ, furnishing a natural geometric framework for the study of Lie bialgebra structures.

In this talk, we investigate Lie bialgebra structures on $\mathfrak{g}$, encoded by 1-cocycles

$$\xi : \mathfrak{g} \longrightarrow \wedge^2 \mathfrak{g},$$

whose transposes define Lie brackets on the dual $\mathfrak{g}^*$. Under a generic non-resonance hypothesis on the Milnor weights, we explicitly determine the $\mathfrak{g}$-invariant subspaces of $\wedge^2\mathfrak{g}$ and $\wedge^3\mathfrak{g}$, and we establish that every such cocycle admits a canonical decomposition of the form

$$\xi = \mathrm{ad}\, r + R,$$

where $r \in \wedge^2\mathfrak{g}$ is a classical r-matrix and R is a linear map satisfying the geometric constraints

$$R(\mathfrak{s} \oplus \mathfrak{z}) \subset \left(\wedge^2 \mathfrak{g}\right)^{\mathfrak{g}}, \qquad R(P_\ell) \subset (\mathfrak{s} \wedge P_\ell) \oplus (\mathfrak{z} \wedge P_\ell).$$

Employing the Big Bracket-Maurer-Cartan formalism together with a multiplicity-free assumption and an identity-type normalization, we show that the co-Jacobi identity splits into two independent conditions:

$$\{R, R\} = 0 \qquad \text{and} \qquad [r, r] + 2\{r, R\} \in \left(\wedge^3 \mathfrak{g}\right)^{\mathfrak{g}}.$$

This decomposition cleanly separates the classical Yang–Baxter contribution from the genuinely non-coboundary part, thereby reducing the full classification problem to the study of normalized cocycles satisfying invariant Schouten-square conditions. In particular, the

quasi-triangular locus is completely determined by the Milnor weights, the distinguished area bivectors ω_ℓ, and the associated subalgebras $\mathfrak{s}_\ell$.

Beyond Poisson-Lie theory, we explain how this cocycle decomposition has significant consequences for local deformation theory. Specifically, it illuminates the structure of character varieties

$$\mathrm{Hom}\Big(\pi_1(S_g),\, G\Big)\Big/G$$

for surface groups [2], and more broadly of representation spaces associated with metric abelian and flat metric Lie groups.

Finally, for the simply connected group $G \simeq S \ltimes N$, we integrate these infinitesimal data to construct explicit multiplicative Poisson tensors on G. On the abelian normal subgroup $N = \exp(\mathfrak{z} \oplus \mathfrak{u})$, the normalized cocycle R integrates in closed form as

$$\Pi_R\Big(\exp u\Big) = R(u) + \tfrac{1}{2}[u, R(u)].$$

This formula yields polynomial Poisson-Lie structures on G, and in the fundamental non-reductive example of the Euclidean motion group $E(2)$ it produces an exact quadratic Poisson tensor, providing a concrete and geometrically transparent illustration of the general theory.

References

[1] A. Bahayou, *Bialgebra structures on flat Lie algebras and their Poisson–Lie groups*, preprint (2026), submitted.

[2] W. M. Goldman, *The symplectic nature of fundamental groups of surfaces*, Adv. Math. **54** (1984), no. 2, 200-225.

[3] J. Milnor, *Curvatures of left invariant metrics on Lie groups*, Adv. Math. **21** (1976), no. 3, 293-329.

Induced Geometric Potential for a Spin-1/2 Particle on a Curved Surface

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Abstract. We study the quantum dynamics of a spin-1/2 particle constrained to a curved surface using the thin-layer quantization formalism introduced by C. R. T. da Costa, in which motion is effectively restricted to the surface by a strong confining potential. By incorporating spin connections into this framework, an additional scalar term arises that modifies the conventional geometric potential. We apply this approach to axially symmetric surfaces, with explicit examples including the cylinder and the sphere, and derive the corresponding energy spectra in each case. Our results highlight the interplay between curvature and spin in determining the quantum behavior of the particle, and provide a consistent framework for analyzing spin-1/2 systems confined to curved geometries.

About delta operator and generating functions

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Abstract.

This talk is devoted to delta operator which is linked to a sequence of binomial type polynomials. First we revisit the family of delta operators and binomial type polynomials. Secondly we give the characterization of generating functions of binomial type polynomials, to end with a method for constructing delta operators followed by some examples.

References

[1] G. C. Rota, D. Kahaner and A. Odlyzko: On the Fondation of Combinatorial Thoery. VIII. Finite Operator Calculus,*Journal of Mathematical Analysis and Applications*, **42**, 684–760 (1962).

[2] L. Comtet: Advanced Combinatorics: The Art of Finite and Infinite Expansions, Revised and enlarged edition. D. Reidel Publishing Company, Dordrecht, 1974.

[3] H. S. Wilf: Generatingfunctionology, Second Edition, Academic Press, 1994.

On n-pre-Lie algebras and dendrification of n-Lie algebras

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Abstract.

The main purpose of this paper is to introduce the notion of n-L-dendriform algebra which can be seen as a dendrification of n-pre-Lie algebras by means of $\mathcal{O}$-operators. We investigate the representation theory of n-pre-Lie algebras and provide some related constructions. Furthermore, we introduce the notion of phase space of a n-Lie algebra and show that a n-Lie algebra has a phase space if and only if it is sub-adjacent to a n-pre-Lie algebra. Moreover, we present a procedure to construct $(n+1)$-pre-Lie algebras from n-pre-Lie algebras equipped with a generalized trace function.

References

[1] M. Aguiar and J.-L. Loday, Quadri-algebras, *J. Pure Applied Algebra*, 191, (2004), 205-221.

[2] J. Arnlind, A. Makhlouf , S. Silvestrov, Ternary Hom-Nambu-Lie algebras induced by Hom- Lie algebras, *J. Math. Phys.* 51, 043515, 11 pp. (2010).

[3] J. Arnlind, A. Makhlouf, S. Silvestrov., Construction of n-Lie algebras and n-ary Hom-Nambu- Lie algebras, *J. Math. Phys.* 52, 123502, 13 pp. (2011).

[4] J. Arnlind, A. Kitouni, A. Makhlouf, S. Silvestrov, Structure and cohomology of 3-Lie algebras induced by Lie algebras. *In Algebra, Geometry and Mathematical physics* (pp. 123-144) (2014). Springer, Berlin, Heidelberg.

[5] H. Awata, M. Li, D. Minic, Yoneya T., On the quantization of Nambu brackets, *J. High Energy Phys.* 2, Paper 13, 17 pp. (2001).

[6] C. Bai, X. Ni, Pre-alternative algebras and pre-alternative bialgebras, *Pacific J. Math.* 248 (2010) 355–390.

Classification of Supermodule Structures and Twistings over Hopf Superalgebras

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Abstract.

Hopf superalgebras and their representation theory play a fundamental role in the study of $\mathbb{Z}_2$-graded algebraic structures and supersymmetry [4]. In this work, we address the classification of supermodule and supermodule superalgebra structures over finite-dimensional Hopf superalgebras [3]. Building on classification results in low dimensions, notably those of Aissaoui and Makhlouf [1], we aim to further clarify the structural properties of these objects.

We also investigate deformation techniques based on twisting and 2-cocycle constructions. In particular, we extend Drinfeld's twisting framework to the $\mathbb{Z}_2$-graded case and study its interaction with supermodule structures, in connection with the approach developed by Giaquinto and Zhang [2]. These methods provide effective tools for constructing new examples and for understanding isomorphism classes in the category of supermodules. We conclude with illustrative examples and perspectives for further developments [3].

This is a joint work with S. Aissaoui and A. Makhlouf.

References

[1] S. Aissaoui and A. Makhlouf, *On Classification of finite dimensional superbialgebras and Hopf superalgebras*, Symmetry, Integrability and Geometry: Methods and Applications SIGMA 10 (2014), 001, 24 pages.

[2] A. Giaquinto and J. Zhang. Bialgebra actions, twists, and universal deformation formulas. *Journal of Pure and Applied Algebra.* 128 (2):133–151, (1998).

[3] H. Moussaoui, S. Aissaoui and A. Makhlouf. Supermodules, Supercomodules and twistings of finite-dimensional Hopf superalgebras. *Proceedings of the Jangjeon Mathematical Society.* 27 (2): 335–370, 2024.

[4] D. Radford D. *Hopf algebras* . World Scientific, Series on Knots and Everythings,. Vol 49 (2012).

Rota-Baxter type operators on trusses and derived structures

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Abstract.

The aim of this talk is to introduce and study the concepts of the Rota-Baxter operator and Reynolds operator within the framework of trusses. Moreover, we introduce and discuss dendriform trusses, tridendriform trusses, and NS-trusses as fundamental algebraic structures underlying these classes of operators. Furthermore, we consider the notions of Nijenhuis operator and averaging operator to trusses, exploring their properties and applications to uncover new algebraic structures.

References

[1] Chtioui, T., Elhamdadi, M., Mabrouk, S., & Makhlouf, A. (2025). Rota-Baxter type operators on trusses and derived structures. arXiv e-prints, arXiv-2504.

[2] Brzeziński T., Lie trusses and heaps of Lie affebras, arXiv preprint arXiv:2203.12975 (2022)

[3] Brzeziński T., *Trusses: Paragons, ideals and modules*, J. Pure Appl. Algebra **224** (2020), 106258.

Cohomology of Acting on Linear Differential Operators and Deformation of $\mathfrak{sl}(2)$ and $\mathfrak{osp}(1|2)$-modules of symbols

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Abstract.

We consider the $\mathfrak{sl}(2)$-module structure on the spaces of symbols of differential operators acting on the spaces of weighted densities. First, we compute the first cohomology spaces $H^1(\mathfrak{osp}(1|2), D_{\lambda,\mu})$ of the Lie superalgebra $\mathfrak{osp}(1|2)$ with coefficients in the superspace $D_{\lambda,\mu}$ of linear differential operators acting on weighted densities on the supercircle S 1—1. The structure of these spaces was conjectured in (Gargoubi et al. in Lett Math Phys 79:5165, 2007). In fact, we prove here that the situation is a little bit more complicated. Second, we compute the necessary and sufficient integrability conditions of a given infinitesimal deformation of this structure and we prove that any formal deformation is equivalent to its infinitesimal part. We study also the super analogue of this problem of deformation getting the same results.

References

[1] I. Basdouri, M. Ben Ammar, *Cohomology of* $\mathfrak{osp}(1|2)$ *Acting on Linear Differential Operators on the Supercercle* $S^{1|1}$. Letters in Mathematical Physics(2007)81:239–251.

[2] I. Basdouri, M. Ben Ammar, *Deformations of* $\mathfrak{sl}(2)$ *and* $\mathfrak{osp}(1|2)$*-modules of symbols.* Acta Mathematica Hungarica, Volume 137, Issue 3, Year 2012.

[3] I. Basdouri, M. Ben Ammar, N. Ben Fraj, M. Boujelbene and K. Kammoun, *Cohomology of the Lie Superalgebra of Contact Vector Fields on* $\mathbb{R}^{1|1}$ *and Deformations of the Superspace of Symbols* . Jour of Nonlinear Math Physics, Vol. 16, No. 4 (2009) 1–37.

[4] I. Basdouri, M. Ben Ammar, B. Dali and S. Omri, *Deformation of* $\mathfrak{vect}(1)$*-Modules of Symbols.* doi:10.1016/j.geomphys.2009.12.002.

[5] F. Berezin, Introduction to superanalysis. In: Mathematical Physics and Applied Mathematics, vol. 9. Reidel Dordrecht (1987)

[6] B. L. Feigin, D. B. Fuchs, Homology of the Lie algebras of vector fields on the line, *Func. Anal. Appl.*, 14 (1980) 201–212.

[7] A. Fialowski, ”Deformations of Lie algebras,” Mat. Sbornyik USSR„ 127 (169), (1985), pp. 476–482; English translation: Math. USSR-Sb., 55, (1986), no. 2, 467–473

[8] A. Fialowski, ”An example of formal deformations of Lie algebras” NATO Conference on Deformation Theory of Algebras and Applications, Il Ciocco, Italy, 1986, Proceedings. Kluwer, Dordrecht, 1988, pp. 375–401

[9] H. Gargoubi, N. Mellouli and V. Ovsienko *Differential Operators on Supercircle: Conformally Equivariant Quantization and Symbol Calculus*, Letters in Mathematical Physics (2007) **79**: 51–65.

[10] P. B. A. Lecomte, *On the cohomology of* $\mathfrak{sl}(n+1;\mathbb{R})$ *acting on differential operators and* $\mathfrak{sl}(n+1;\mathbb{R})$*-equivariant symbols,* Indag. Math. NS. 11 (1), (2000), 95 114.

[11] D. Leites, Supermanifold Theory. (in Russian) USSE Academy of Science Petrozavodsk (1983)

[12] A. Nijenuis, R. W. Richardson Jr., *Deformations of homomorphisms of Lie groups and Lie algebras*, Bull. Amer. Math. Soc. **73** (1967), 175–179.

[13] V. Ovsienko, C. Roger, *Deforming the Lie algebra of vector fields on* S^1 *inside the Poisson algebra on* $\dot{T}^*S^1$, Comm. Math. Phys., **198** (1998) 97–110.

[14] R.W. Richardson, *Deformations of subalgebras of Lie algebras*, J. Diff. Geom. , 3, (1969), 289-308.

When Geometry Dictates Physics: Defining a Particle in Cosmology

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Abstract.

In quantum field theory on a curved spacetime, the absence of Poincaré invariance raises fundamental ambiguities regarding the selection of a unique vacuum and even the definition of particles. This issue becomes more subtle in the context of an expanding universe, where the time dependence of the scale factor introduces an inherent non-uniqueness in the choice of field variables to be quantized. We show that reparameterizations of the scalar field correspond to distinct foliations of the symplectic phase space, reflecting the underlying geometry and dynamical structure of the system. Although classically indistinguishable, these transformations generally lead to inequivalent quantum theories. Imposing the preservation of the canonical structure together with adiabatic evolution significantly constrains the allowed choices. Within this framework, the concept of a particle in an expanding universe is therefore not arbitrary, but emerges from a rigorous geometric and dynamical selection dictated by the internal consistency of the quantum formalism on curved spacetime.

Joint work with Loubna Cheriet

On the second $\mathfrak{osp}(1|2)$-relative cohomology of the Lie superalgebra of contact vector fields on $\mathcal{C}^{1|1}$

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Abstract.

Let $\mathcal{K}(1)$ be the Lie superalgebra of contact vector fields on the (1,1)-dimensional complex superspace; it contains the Möbius superalgebra $\mathfrak{osp}(1|2)$. We classify $\mathfrak{osp}(1|2)$-invariant superanti-symmetric binary differential operators from $\mathcal{K}(1) \wedge \mathcal{K}(1)$ to $\mathfrak{D}_{\lambda,\mu}$ vanishing on $\mathfrak{osp}(1|2)$, where $\mathfrak{D}_{\lambda,\mu}$ is the superspace of linear differential operators acting on the superspaces of weighted densities. This result allows us to compute the second differential $\mathfrak{osp}(1|2)$-relative cohomology of $\mathcal{K}(1)$ with coefficients in $\mathfrak{D}_{\lambda,\mu}$.

References

[1] Basdouri, I., Ben Ammar, M., Ben Fraj, N., Boujelbene, M., Kammoun, K; "Cohomology of the Lie superalgebra of contact vector fields on $\mathbb{K}^{1|1}$ and deformations of the superspace of symbols", *J. Nonlinear Math. Phys*, **16** 373 (2009).

[2] Belghith, N; Ben Ammar, M; Ben Fraj, N: "Differential operators on the weighted densities on the supercircle $S^{1|1}$". *Studia Scientiarum Mathematicarum Hungarica* **52:4**, 477–503 (2015).

[3] Ben Ammar, M., Boujelbene, M; "$\mathfrak{sl}(2)$-Trivial deformations of $Vect_P(\mathbb{R})$-modules of symbols". SIGMA **4**, 065 (2008).

[4] Ben Fraj, N., Abdaoui, M., Raouafi, H; "On $\mathfrak{osp}(2|2)$-relative cohomology of the Lie superalgebra of contact vector fields and deformations", *J. Geom. Phys* **125**, (33-48)(2018).

[5] Ben Fraj, N., Abdaoui, M., Raouafi, H; "On $\mathfrak{osp}(1|2)$-relative cohomology of the Lie superalgebra of contact vector fields on $\mathbb{R}^{1|1}$", *Int. Jour. Geom. Methods. Mod. Phys* **14. 2**, 1750022(1-29)(2017).

[6] Bouarroudj, S; "Cohomology of the vector fields Lie algebras on $\mathbb{RP}^1$ acting on bilinear differential operators", *Int. Jour. Geom. Methods. Mod. Phys*, **2:1** 23-40 (2005).

[7] Bouarroudj, S; "Projective and conformal Schwarzian derivatives and cohomology of Lie algebras vector fields related to differential operators", *Int. Jour. Geom. Methods. Mod. Phys,* **3:4** 667-696 (2006).

[8] Bouarroudj, S., and Ovsienko, V; "Three cocycle on Diff(S^1) generalizing the Schwarzian derivative," *Int.Math.Res.Notices*, 25-39 (1998).

[9] Bouarroudj, S; "On $\mathfrak{sl}(2)$-relative Cohomology of the Lie algebra of vector fields and differential operators", *J. Nonlinear Math. Phys*, **14:1** 1–29 (2007).

[10] Conley, C. H; "Conformal symbols and the action of Contact vector fields over the superline", *J. Reine Angew. Math,* **633** 115–163 (2009).

[11] Fuks [Fuchs] D. B; "Cohomology of infinite-dimensional Lie algebras", Plenum Publ. New York, 1986.

[12] Gargoubi, H., Mellouli, N., and Ovsienko, V; "Differential operators on supercircle: conformally equivariant quantization and symbol calculus", *Lett. Math. Phys,* **79** 51-–65 (2007).

[13] Gargoubi, H., and Ovsienko, V; Supertransvectants and symplectic geometry, *Internat. Math. Res. Notices*, 2008 ID rnn021; `arXiv`: 0705.1411v1 [math-ph].

[14] Gieres, F., Theisen, S; "Superconformally covariant operators and super W-algebras", *J. Math. Phys.* **34** 5964–5985 (1993).

[15] Huang, W. J; "Superconformal covariantization of pseudodifferential opera- tor on $(1 \mid 1)$-superspace and classical $N = 2W$-superalgebras",*J. Math. Phys.* **35:5** 2570–82 (1994).

[16] Kassel, C., "Quantum Groups", Graduate Texts in Mathematics Vol. 155 (Springer-Verlag, New York, 1995).

[17] Lecomte, P. B. A., and Ovsienko, V; "Cohomology of the vector fields Lie algebra and modules of differential operators on a smooth manifold", *Compositio Mathematica* **124:1** 95-–110 (2000).

[18] Nijenuis, A., Richardson, Jr. R. W; "Deformations of homomorphisms of Lie groups and Lie algebras", *Bull. Amer. Math. Soc*, **73** 175–179 (1967).

[19] Ovsienko, V., Roger, C; "Extension of Virasoro group and Virasoro algebra by modules of tensor densities on S^1", *Funct. Anal. Appl*, **31:4** (1996).

Order of an element in the Hom-groups and some applications

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Joint work with Sergei Silvestrov

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Abstract.

This presentation defines the order of an element in Hom groups and some important properties useful for classifying Hom groups. This addresses the unanswered questions posed in the article [9]. Furthermore, some applications are proposed, for example, Cauchy's theorem and Sylow's theorem, which demonstrate the importance of non-associative algebraic structures.

References

[1] N. Aizawa and H. Sato, q-deformation of the Virasoro algebra with central extension, Phys. Lett. B 256 (1991), 185-190.

[2] I. Basdouri., S. Chouaibi., A. Makhlouf., E. Peyghan., Free Hom-groups, Hom-rings and Semisimple modules, *arXiv:2101.03333v1,* (2020).

[3] H.Bourmel, Z.Chebel and H.Adimi, Ordered Hom-groups based on Hom-compatibility, Rendiconti del Circolo Matematico di Palermo Series 2, Volume 72, (2023), pages 3377–3398.

[4] M. Chaichian, P. Kulish, and J. Lukierski, q-deformed Jacobi identity, q-oscillators and q-deformed infinite-dimensional algebras, Phys. Lett. B 237 (1990), 401-406.

[5] Z. Chebel, H. Adimi and H. Bourmel, Hom-actions and Class Equation for Hom-groups. Accepted for publication in Journal of Geometry and Physics (2024).

[6] T. L. Curtright and C. K. Zachos, Deforming maps for quantum algebras, Phys.Lett. B 243 (1990), 237-244.

[7] L. Chen,. T. Feng., Y. Ma., R. Saha.,H. Zhang., On Hom-Groups and Hom-Group actions, Acta Mathematica Sinica, Volume 39, (2023), pages 1887–1906.

[8] M. Hassanzadeh. Hom-groups, Representations and homological algebra, Colloq. Math. 158 (2019), no. 1,21–38.

[9] M. Hassanzadeh. Lagrange's theorem For Hom-Groups, Rocky Mountain J. Math. 49 (2019), no. 3, 773–787.

[10] J. T. Hartwig, D. Larsson and S. D. Silvestrov, Deformations of Lie algebras usin σ-derivations, J. Algebra 295 (2006), no. 2, 314-361.

[11] C. Laurent-Gengoux, A. Makhlouf, and J. Teles, Universal algebra of a Hom-Lie algebra and group like elements, Journal of Pure and Applied Algebra, Volume 222, Issue 5, (2018), P. 1139–1163.

[12] Jun Jiang, Satyendra Kumar Mishra, Yunhe Sheng, Hom-Lie algebras and Hom-Lie groups, integration and differentiation, arXiv:1904.06515.

[13] A. Makhlouf and S. Silvestrov, Hom-algebra structures, J. Gen. Lie Theory Appl. **2**(2), 51–64 (2008).

[14] A. Makhlouf and S. Silvestrov, Notes on formal deformations of Hom-associative and Hom-Lie algebras, Forum Math. **22**(4), 715–759 (2021).

[15] D. Yau, Enveloping algebras of Hom-Lie algebras, J. Gen. Lie Theory Appl. 2 (2008), no. 2, 95–108.

Results on some $D(-1;1)$-Diophantine triples

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Abstract.

Let $m \geq 2$ and n be integers. A set of m distinct positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple with property $D(n)$, or simply a $D(n)$-m-tuple, if for every pair $1 \leq i < j \leq m$, the expression $a_i a_j + n$ is a perfect square. The size of such sets has been an important area of research. A. Dujella showed that if $n \not\equiv 2 \pmod 4$ and $n \notin S := \{-4, -3, -1, 3, 5, 8, 12, 20\}$, then there exists at least one $D(n)$-quadruple [3]. The nonexistence of a $D(1)$-quintuple was established in [4], and further results on the nonexistence of $D(4)$-quintuples were shown in [5]. In [1], N. C. Bonciocat et al. proved the nonexistence of a $D(-1)$-quadruple, which, according to [3, Remark 3], implies the nonexistence of a $D(-4)$-quadruple.

Although no $D(-1)$-triple $\{a, b, c\}$ with $a < b < c$ can be extended to a $D(-1)$-quadruple, there exists an integer $d = d^{\pm}$ such that $ad + 1$, $bd + 1$, and $cd + 1$ are all perfect squares, where

$$d^{\pm} = 2abc - (a + b + c) \pm 2\sqrt{(ab - 1)(ac - 1)(bc - 1)}$$

(see Lemma 3 in [2]). Consequently, a set $\{a, b, c; d\}$ of positive integers is said to possess the property $D(-1; 1)$ if $\{a, b, c\}$ is a $D(-1)$-triple and each of $ad + 1$, $bd + 1$, and $cd + 1$ is a perfect square.

In this paper I will focus on some results on the $D(-1)$-Diophantine triple induced by the $D(-1)$-pair $\{1, 5\}$. This is a jointly work with Y. Briedj.

References

[1] N. C. Bonciocat, M. Cipu and M. Mignotte, *There is no Diophantine $D(-1)$-quadruple*, J. London Math. Soc. **105.1** (2022), 63–99.

[2] A. Dujella, *On the size of Diophantine $m-$tuples*, Math. Proc. Cambridge Philos. Soc. **132** (2002), 23–33.

[3] A. Dujella, *Generalization of a problem of Diophantus*, Acta Arith., **65.1** (1993), 15–27.

[4] B. He, A. Togbé and V. Ziegler, *There is no Diophantine quintuple*, Trans. Amer. Math. Soc. **371.9** (2019), 6665–6709.

[5] M. B. Trebješanin and A. Filipin, *Nonexistence of $D(4)$-quintuples*, J. Number Theory **194** (2019), 170–217.

On solvability and nilpotence of n-Hom-Lie algebras

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Abstract.

We present a generalization of derived series, central descending series, solvability and nilpotence for the case of n-Hom-Lie algebras. In [1], the author introduces several different derived series and central descending series for n-Lie algebras, namely k-derived series and k-central descending series, where k is between 2 and n. This possibility arises from the higher arity of the bracket and it allows to have multiple different algebraic invariants of that type. We extend these notions to n-Hom-Lie algebras and study their properties. The terms of the k-derived series and k-central descending series are not necessarily ideals as in the classical case, we study whether some weaker conditions are still satisfied and under which conditions the properties of the classical case are conserved. We also present a few examples to illustrate.

References

[1] Kasymov, Sh. M.: Theory of n-Lie algebras, Algebra and Logic, **26**, 155-166 (1987)

10 Communications: Titles and Abstracts

Averaging Poisson algebra to noncommutative Leibniz-Poisson algebras

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Abstract.

A noncommutative Leibniz–Poisson algebra is a natural generalization of a Poisson algebra, obtained by relaxing the commutativity condition. The main objective of this work is to construct such structures by applying an averaging operator to a given Poisson algebra. More precisely, we show how the averaging process transforms the original Poisson structure into a noncommutative Leibniz–Poisson algebra. Furthermore, this approach is applied to a specific class of Poisson algebras, and the properties of the resulting structures are investigated.

Poisson algebras play a fundamental role in several areas of mathematics and mathematical physics, particularly in Hamiltonian mechanics, deformation theory, and symplectic geometry. They combine two compatible structures: a commutative associative algebra and a Lie algebra structure linked by the Leibniz rule.

In recent years, there has been growing interest in noncommutative generalizations of classical algebraic structures, motivated by developments in noncommutative geometry and quantum algebra. Among these generalizations, Leibniz–Poisson algebras arise as a natural extension where the Lie bracket is replaced by a Leibniz bracket, and the commutativity assumption is removed. In this paper, we propose a systematic method to derive noncommutative Leibniz–Poisson algebras from classical Poisson algebras by means of an averaging operator. We then apply this construction to a particular class of Poisson algebras.

Definition 10.1. *Non commutative Leibniz-Poisson algebra (NLP) A non commutative Leibniz-Poisson algebra $(A, *, \{\cdot,\cdot\})$ is an associative algebra with Leibniz bracket $\{\cdot,\cdot\}$ satisfying the left identity together with a compatibility condition :*

$$\{a, b * c\} = \{a, b\} * c + b * \{a, c\} \quad \forall a, b, c \in A.$$

Definition 10.2. *A linear map $T : A \to A$ is an averaging operator on the algebra A if : $T(a)T(b) = T(T(a)b) = T(aT(b)), \quad \forall a, b \in A.$*

Averaging Poisson Algebras to Noncommutative Leibniz-Poisson Algebras

Theorem 10.3. *Let $(A, \cdot, [\cdot,\cdot])$ be a Poisson algebra, and $T : A \to A$ be an averaging operator satisfying :*

$$[T(a), T(b)] = T([T(a), b])$$

Then $(A, *_T, \{\cdot,\cdot\}_T)$ *is a non commutative Leibniz-Poisson algebra.*

$$a *_T b = T(a) \cdot b \quad \textit{and} \quad \{a, b\}_T = [T(a), b].$$

References

[1] "Degenerations of Poisson algebras", A. Hani, O. Amir, F. González and C. Martín, Journal of Algebra and its Applications. volume(24), number(03), 2025

[2] "Hom-associative algebras, Admissibility and Relative averaging operators", S. Braiek, T. Chtioui, S. Mabrouk and M. Elhamdadi, arXiv preprint arXiv2311.08407. 2023

[3] "Noncommutative Leibniz–Poisson algebras", J.M. Casas and T. Datuashvili, Communications in Algebra. volume(34), number(07), 2006

Global Dynamics and Memory-Driven Diffusion in Semilinear Nonlocal Models of Mathematical Physics

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Abstract.

The qualitative study of nonclassical evolution equations has attracted considerable attention in recent decades, motivated by their appearance in viscoelasticity, heat conduction with memory, and anomalous diffusion phenomena. A central challenge in this field is to understand how the competition between dispersive effects, fractional-order operators, and hereditary memory terms shapes the long-time behaviour of solutions. The present work addresses this challenge for the Cauchy problem

$$\partial_t u \;-\; \Delta\,\partial_t u \;+\; (-\Delta)^{\theta} u \;-\; g * \Delta\, u \;=\; f(u), \qquad t>0, \quad x\in\mathbb{R}^n,$$

with initial datum $u(x,0)=u_0(x)$, where $0<\theta\le 1$, g is a non-negative summable memory kernel, and the nonlinearity satisfies $f(u)=\mathcal{O}(|u|^{\alpha})$ as $|u|\to 0$ for some $\alpha>1$.

The equation combines three dissipative mechanisms acting on different frequency regimes: the pseudo-parabolic damping $-\Delta\partial_t u$, which regularises high-frequency oscillations; the fractional diffusion $(-\Delta)^{\theta}$, which governs intermediate-frequency energy transfer; and the convolution term $g*\Delta u$, which encodes the history of the solution and introduces a nonlocal-in-time dissipation. Disentangling their individual contributions and quantifying their collective effect constitutes the main mathematical novelty of this study.

Strategy and main results. Our approach rests on three pillars.

(i) Lyapunov analysis in Fourier space. We introduce the frequency-localised energy functional

$$\mathcal{E}(t,\xi) \;=\; \left(1+|\xi|^2\right)|\hat{u}(t,\xi)|^2 \;+\; |\xi|^2\int_0^{\infty}\mu(s)\,|\hat{\eta}^t(s,\xi)|^2\,ds,$$

where $\hat{\eta}^t$ is the memory history variable and $\mu=-g'$ is the associated kernel. A direct computation yields the dissipation identity

$$\frac{d}{dt}\mathcal{E}(t,\xi) \;+\; 2|\xi|^2|\hat{u}(t,\xi)|^2 \;+\; |\xi|^2\int_0^{\infty}\mu(s)\,|\hat{\eta}^t(s,\xi)|^2\,ds \;=\; 0,$$

which shows that $\mathcal{E}$ is a genuine Lyapunov functional for every fixed frequency ξ.

(ii) Pointwise decay and the dissipation-rate symbol. A careful low-frequency/high-frequency splitting leads to the pointwise exponential bound

$$\mathcal{E}(t,\xi) \;\le\; C\,\mathcal{E}(0,\xi)\,\exp\Big(-c\,\rho(\xi)\,t\Big), \qquad \rho(\xi) = \frac{|\xi|^{2\theta}}{1+|\xi|^2}.$$

The symbol $\rho(\xi)$ reflects the balance between the fractional operator at intermediate frequencies and the pseudo-parabolic term at high frequencies; it is precisely this symbol that dictates the sharp polynomial decay rate after integration over ξ.

(iii) Global Sobolev estimates and optimal decay. Integrating the pointwise bound against Sobolev weights, we obtain, for $s \ge 1$ and $u_0 \in H^s(\mathbb{R}^n)$,

$$\|u(t)\|_{H^s}^2 \;+\; \int_0^t \|\nabla u(\tau)\|_{H^{s-1}}^2\,d\tau \;\le\; C\,\|u_0\|_{H^s}^2.$$

When additionally $u_0 \in L^1(\mathbb{R}^n)$, the low-frequency contribution of the linearised solution decays algebraically, yielding the sharp derivative estimates

$$\|\nabla^r u(t)\|_{H^{s-r}} \;\le\; C\,(1+t)^{-\frac{n}{4\theta}-\frac{r}{2\theta}}, \qquad r = 0,1,\dots,s,$$

which are optimal in the sense that they match the decay of the heat kernel associated with $(-\Delta)^\theta$.

Semilinear problem. For the full nonlinear problem, the term $f(u)$ is treated as a perturbation inside a Banach contraction argument performed in a time-weighted Sobolev space. Under a smallness condition on $\|u_0\|_{H^s\cap L^1}$, and provided α is large enough relative to n and θ, we establish the existence of a unique global solution

$$u \;\in\; C\Big([0,\infty);\,H^s(\mathbb{R}^n)\Big)$$

that inherits, without loss, the same optimal decay rates as the linear problem. This confirms that the memory and fractional-diffusion mechanisms dominate the nonlinear interaction in the large-time regime.

Significance. The results presented here unify and extend several contributions in the literature on strongly damped wave equations, pseudo-parabolic models, and fractional diffusion. They provide a rigorous framework for quantifying how hereditary memory accelerates or retards energy decay, and they open the way to further investigations in bounded domains, variable-exponent settings, and stochastic perturbations.

References

[1] C. M. Dafermos, *Asymptotic stability in viscoelasticity*, Arch. Ration. Mech. Anal. **37** (1970), 297–308.

[2] V. Pata and M. Squassina, *On the strongly damped wave equation with memory*, Comm. Math. Phys. **253** (2005), 511–533.

[3] G. Karch, *Selfsimilar profiles in large time asymptotics of solutions to damped wave equations*, Studia Math. **143** (2000), 175–197.

[4] C. Miao and G. Xu, *Global well-posedness for semilinear fractional diffusion equations*, J. Differential Equations **252** (2012), 4625–4657.

[5] R. Racke, *Lectures on Nonlinear Evolution Equations*, Birkhäuser/Springer, Basel, 1992.

On the D-isometric deformation of Kenmotsu manifolds and biharmonic maps

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Abstract.

In this paper, we introduce a D-isometric deformation of almost contact metric manifold where we give some new results of this type of deformation, the harmonicity and the biharmonicity of the identity map are characterized by using this deformation.

Keywords: Harmonic and biharmonic maps, Almost contact metric manifolds, Kenmotsu manifolds, D -isometric deformation.

References

[1] A. Balmus, "Biharmonic properties and conformal changes," *An. Ştiint. Univ. Al.I. Cuza Iaşi Mat. (N.S.)*, vol. 50, pp. 367–372, 2004.

[2] G. Beldjilali and M. A. Akyol, "D-homothetically deformed Kenmotsu metric as a Ricci soliton," *Annales Mathematicae Silesianae*, vol. 33, pp. 143–152, 2019.

[3] G. Y. Jiang, "2-harmonic maps and their first and second variational formulas," *Chinese Ann. Math. Ser. A*, vol. 7, pp. 389–402, 1986.

[4] A. De, "On Kenmotsu manifold," *Bulletin of Mathematical Analysis and Applications*, vol. 2, no. 3, pp. 1–6, 2010.

On a problem of two immiscible fluids flows.

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Abstract.

We consider a problem of two flows of immiscible fluids flowing in an infinite channel in 2D over an obstacle. The flows are irrotational, stationary and the fluids are ideals and incompressible. The main unknowns of the problem are the interface between the two fluids and the free surface of the second flow. The problem is non linear, using the perturbation stream functions, we linearize the problem then we use the Lax-Milgram's theorem to prove the existence and the uniqueness of the solution of the problem.

References

[1] W. Choi and R. Camassa, Weakly nonlinear internal waves in a two-fluid system, J. Fluid Mech., **Vol.313** (1996), 83-103.

[2] N. Foukroun, R. Ait-Yahia-Djouadi, D. Hernane-Boukari, Solvability of a supercritical free surface flow problem under gravity-capillarity, Applicable Analysis (2018) **Vol 97 N°10**, 1701-1716. DOI: 10.1080/00036811.2017.1335395

[3] N. Foukroun, D. Hernane-Boukari, R. Ait-Yahia-Djouadi, Solvability of a three dimensional free surface flow problem over an obstacle, Applicable Analysis (2022) **Vol 101, N°2**, 554-573. DOI: 10.1080/00036811.2020.1754401

[4] Hishyar Kh. Abdullah, On the two immiscible fluids in contact with each other on a solid boundary, Applied Mathematical Sciences, **Vol. 6, no. 123**, (2012), 6137 - 6146.

[5] T. Iguchi, Two-Phase Problem for Two-Dimensional Water Waves of Finite Depth, Mathematical Models and Methods in Applied Sciences, **Vol. 07 (06)**, (1997), 791-821.

[6] A. C. King, M.I.G. Bloor , Free streamline flow over curved topography, Quarterly Of applied Mathematics, **XLVIII (2)**, Univ. Nottingham, dep. mathematics,Royaume-Uni(1990), 281-293.

[7] D. Lannes, A stability criterion for two-fluid interfaces and applications, Archive for Rational Mechanics and Analysis, **208**, (2013), 481-567.

[8] C. Titri-Bouadjenak · D. E. Teniou, Flows of two immiscible fluids over an obstacle, SeMA **63**, (2014), 1-26. DOI 10.1007/s40324-014-0009-6

Existence and Properties of Primitive Periods for a New Class of Generalized Fibonacci Functions

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Abstract.

We introduce a new class of generalized Fibonacci functions and prove the existence of a primitive period modulo any integer $m > 1$. Fundamental properties of these periods are established, including a divisibility criterion and a formula for composite moduli. Explicit symmetry relations are derived for a special family. Our results unify and extend previous work on periods of Fibonacci functions.

Keywords
Fibonacci function, generalized Fibonacci function, period, primitive period, modular arithmetic.

Main Results

1. **Theorem 01. (Existence of a period)** For any $m > 1$, there exists ℓ with $1 \leq \ell \leq m^{\alpha_1+\cdots+\alpha_k}$ such that $f(n+\ell) \equiv f(n) \pmod{m}$ for all integers n.

2. **Theorem 02. (Divisibility property)** ℓ is a period of f modulo m if and only if $\ell_f(m) \mid \ell$.

3. **Theorem 03. (Period for composite modulus)** If $\gcd(n, m) = 1$, then $\ell_f(nm) = \operatorname{lcm}(\ell_f(n), \ell_f(m))$.

4. **Theorem 04. (Symmetry for a special case)** For $(k : (\lambda, \alpha), (1, 0), \ldots, (1, 0), (\gamma, \alpha))$ with $\alpha \geq 2$ and $f(n) = 0$ for $0 \leq n \leq 2\alpha - 2$,

$$f(p) = \begin{cases} \gamma f(-p + 2\alpha - 2), & p \not\equiv \alpha - 1 \pmod{2\alpha} \\ -f(-p + 2\alpha - 2), & p \equiv \alpha - 1 \pmod{2\alpha} \end{cases}$$

for all $p \geq \alpha$.

Conclusion

We proved the existence of primitive periods modulo m for the $(k:(\lambda_i,\alpha_i))$-step Fibonacci functions, generalizing previous results. Determining explicit formulas for these periods remains open for future work.

References

[1] M. Elmore, *Fibonacci functions*, Fibonacci Q, 5(4), 371-382, (1967).

[2] T. Kochy, *Fibonacci and Lucas numbers with applications*, Pure Appl. Math., Wiley-Intersci. Ser. Texts Monogr. Tracts, New York, NY: Wiley, (2001).

[3] K. Paikhlaew, S. Kerdmongkon, N. Nawapongpipat, Y. Tongron, *The Symmetries of* $(k:\alpha_1,\alpha_2,...,\alpha_k)$*-step Fibonacci Functions*, ASM Sc. J, 18, 1-7, (2023).

[4] N. Thongngam, and R. Chinram, *Periods of Fibonacci Functions Modulo* m, Thaksin University Journal, 22(2), 53-58, (2019).

[5] W. R. Spickerman, *A note on Fibonacci functions* Fibonacci Q., 8, 397-401, (1970).

[6] Y. Tongron, S. Kerdmongkon, *Periods of k-step Fibonacci Functions Modulo* m, Songklanakarin J. Sci. Technol, 44(2), 323-331, (2022).

On p-adic differential operator

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Abstract.

Through a differential operator L, We study a "mod p differential system" $A(y) = 0$ on a disk D that is over a p-adic disk D^*. In this talk, we present the evaluation of index of such differential system. Also we will measures irregularity of such p-adic differential system in a disk. This document proposes a method to connect local and global indices using the Mittag-Leffler theorem. The proposed method aims to enhance our understanding of the interplay between local solutions and global behavior in p-adic differential equations. p-adic differential equations behave very differently from classical ones. This is mainly due to the non-archimedean nature of the field, which affects convergence and the behavior of solutions. As a result, special methods are required, especially when studying singular points and computing the index of operators. This study highlights the importance of understanding the unique properties of p-adic differential equations, particularly in relation to their singular points and irregularities in solutions.
The objectives are:

Explain how to calculate the index of a p-adic differential operator using complex analysis methods.

Examine how regular and irregular singularities affect the indices.

Analyze existing approaches such as the Adolhpson method and introduce new analytical tools. The p-adic differential operator, an important concept in functional and numerical analysis, plays a prominent role in the study of dynamical systems and differential equations in the non Archimedean sense. Calculating the index of a differential operator shows the number of independent solutions of a differential equation. This helps to better understand the operator, particularly if there is a singularity present. This study deals with the measurement of irregularity in a disk. The effect of regular and irregular singularities on the index of the operator is analyzed. The calculating the index, including Adolphson's method, are considered and new, more precise and adaptable analytical techniques are proposed. In summary, our work aims to bridge local and global perspectives in p-adic differential equations, providing a framework for future research in this area.The interplay between local and global indices is crucial for understanding the structure of solutions in p-adic differential equations.

References

[1] M. S. Rezaoui, *Indice polynomial d'une matrice d'opérateurs différentiels*, Comptes Rendus de l'Académie des Sciences - Series I - Mathematics, Vol. 332 (2001), pp. 505–508.

[2] P. Robba, *Indice d'un opérateur différentiel p-adique IV. Cas des systèmes. Mesure de l'irrégularité dans un disque*, Annales de l'institut Fourier, Vol. 35 (1985), pp. 13–55.

[3] A. Adolphson, *An Index Theorem for p-adic Differential Operators*, Transactions of the American Mathematical Society, Vol. 216 (1976), pp. 279–293.

Qualitative and Bifurcation Analysis of a Reversible Quartic Polynomial Differential System

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Abstract.

We consider the class of planar quartic polynomial differential systems given by

$$\dot{x} = -y, \qquad \dot{y} = x + ax^4 + bx^2y^2 + cy^4,$$

where $a, b, c \in \mathbb{R}$ and $a^2 + b^2 + c^2 \neq 0$. These systems are symmetric with respect to the x-axis and, under suitable conditions, may admit a center at the origin. Systems of this form constitute a natural extension of cubic and quadratic reversible systems and provide a rich class for studying global phase portraits and bifurcation phenomena. In this study, we focus on the particular case $c = 0, \quad b = 1$, which leads to the one-parameter family

$$\dot{x} = -y, \qquad \dot{y} = x + ax^4 + x^2y^2, \qquad a \in \mathbb{R}.$$

Our aim is to describe the qualitative behavior of this system for all real values of a, including the classification of its global phase portraits in the Poincaré disc, and to investigate the bifurcation of small-amplitude limit cycles under quartic perturbations.

The linearization of the system at the origin has purely imaginary eigenvalues $\pm i$, which indicates the presence of a center. Moreover, the system is reversible under the transformation $(x, y, t) \mapsto (x, -y, -t)$, which preserves the center at the origin for all values of a. When $a \neq 0$, an additional finite singular point arises on the x-axis at $(-a^{-1/3}, 0)$. A direct computation of the Jacobian matrix at this point shows that it is a hyperbolic saddle. If $a = 0$, the origin remains the only finite singularity of the system. These results establish the local phase portrait around finite singular points and guarantee that all nearby periodic orbits are organized around the center at the origin.

To understand the global phase portrait, we employ Poincaré compactification, which maps the plane into the closed unit disc and allows the study of singularities at infinity. The structure of infinite singular points depends on the sign of a:

Case $a < 0$: Two semi-hyperbolic saddle-nodes appear at infinity. Each saddle-node has two hyperbolic sectors and one parabolic sector, whose orientation is fully determined by the vector field.

Case $a = 0$: The infinite singularity reduces to a linearly zero critical point, which is shown through blow-up analysis to be topologically equivalent to a stable node.
Case $a > 0$: Semi-hyperbolic saddle-nodes also appear, but in distinct locations relative to the Poincaré disc compared with the $a < 0$ case, and their sectorial decomposition is completely characterized.
This analysis demonstrates that the global phase portrait depends entirely on the sign of a, resulting in a finite number of topologically distinct configurations. The combination of finite and infinite singularities provides a comprehensive qualitative picture of the system's dynamics.
Combining local analysis, compactification, and perturbation methods, we obtain a full qualitative classification of the system for all $a \in \mathbb{R}$, highlighting how symmetry and non-linearities shape global dynamics and limit cycle behavior.

References

[1] N. N. Bautin, On the number of limit cycles which appear with the variation of coefficients from an equilibrium position of focus or center type, *Mat.Sbornik*, 30 (1952), 181–196; *Amer. Math. Soc. Transl.*, 100 (1954), 1–19.

[2] R. Benterki and J. Llibre, Centers and limit cycles of polynomial differential systems of degree 4 via averaging theory, *J. Comput. Appl. Math.*, 313 (2017), 273–283.

[3] C. A. Buzzi, J. Llibre and J. C. Medrado, Phase portraits of reversible linear differential systems with cubic homogeneous polynomial nonlinearities having a non-degenerate center at the origin, *Qual. Theory Dyn. Syst.*, 7 (2009), 369–403.

On low-degree algebraic points arising from morphisms between generalized Fermat curves and their quotients $\mathcal{F}_{\gamma}^{m}(p)$ and $\mathcal{C}_{\eta,\lambda}^{m}(p)$

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Abstract.

In this work, we provide an explicit description of algebraic points of degree at most 3 on the quotient of the Fermat curve $\mathcal{C}_{r,r}^{1}(p)$ with $r \in \left\{1, \ldots, \frac{p-1}{2}\right\}$. We then extend this result via the morphism $\varphi_{r,s}^{\alpha,\beta}$ between quotient Fermat curves $\mathcal{C}_{\eta,\lambda}^{m}(p)$, where $(\eta, \lambda) \in \{1\} \otimes \left\{1, \frac{p-1}{2}, p-2\right\}$. Furthermore, we use the isomorphisms $\Phi_{m,p}$, $\Psi_{m,p}$, and $\Upsilon_{m,p}$ to relate these curves to Fermat-type curves $\mathcal{F}_{\gamma}^{m}(p)$, with $\gamma \in \{1, 2\}$. Our approach relies on classical methods as well as fundamental results such as Clifford's theorem and the Riemann–Roch theorem. These tools allow us to describe a basis of the associated linear system and to apply the Abel–Jacobi theorem in order to determine the explicit form of the function associated with a given rational divisor. This ultimately enables us to characterize the structure of these points together with the constraints that define them.

References

[1] O. Sall, Points algébriques de petit degré sur les courbes de Fermat, C. R. Acad. Sci. Paris 330 (2000), 67–70.

[2] T. Top, O. Sall, Points algébriques de degrés au plus 12 sur la quintique de Fermat, Acta Arith. 169 (2015), 385–395.

[3] Flynn, E. V. and Wetherell, J. L., Finding rational points on bielliptic genus 2 curves, Manuscripta Mathematica, 100 (1999), 519–533.

[4] Lang, Serge, Hyperbolic and Diophantine analysis, Bull. Amer. Math. Soc., 14 (1986), 159–205.

[5] Mordell, L. J., On the rational solutions of the indeterminate equations of the third and fourth degrees, Proc. Cambridge Philos. Soc., 21 (1922), 179–192.

[6] Silverman, Joseph H., The Arithmetic of Elliptic Curves, Springer, 1986.

[7] Lang, Serge, Fundamentals of Diophantine Geometry, Springer, 1983.

[8] Mumford, David, Abelian Varieties, Oxford University Press, 1970.

[9] Debarre, Olivier, Complex Tori and Abelian Varieties, American Mathematical Society, (2005).

[10] Arbarello, Enrico and Cornalba, Maurizio and Griffiths, Phillip and Harris, Joe, Geometry of Algebraic Curves, Vol. I, Springer, (1985).

[11] Arbarello, Enrico and Cornalba, Maurizio and Griffiths, Phillip, Geometry of Algebraic Curves, Springer, (1985).

[12] Gross, Benedict H. and Rohrlich, David E., Some results on the Mordell–Weil group of the Jacobian of the Fermat curve, Inventiones Mathematicae, 44 (1978), 201–224.

[13] Rohrlich, David E., Points at infinity on the Fermat curves, Inventiones Mathematicae, 39 (1977), 95–127.

[14] Tzermias, Pavlos, The group of rational points on the Jacobian of the Fermat curve, Journal für die reine und angewandte Mathematik, 511 (1999), 165–178.

[15] Namba, Makoto, Families of Meromorphic Functions on Compact Riemann Surfaces, Springer, (1979).

[16] Griffiths, Phillip and Harris, Joe, Principles of Algebraic Geometry, Wiley, (1994).

[17] Miranda, Rick, Algebraic Curves and Riemann Surfaces, American Mathematical Society, (1995).

[18] Klassen, Michael and Tzermias, Pavlos, Algebraic points of low degree on the Fermat quintic, Acta Arithmetica, 82 (1997), 393–402.

[19] Sall, Abdoul Aziz, Points algébriques de degré borné sur les courbes de Fermat, Acta Arithmetica, 95 (2000), 1–20.

[20] Top, Jaap and Sall, Abdoul Aziz, Algebraic points of bounded degree on Fermat curves, Mathematical Proceedings of the Cambridge Philosophical Society, 159 (2015), 1–18.

[21] Flynn, E. V. and Wetherell, J. L., Finding rational points on bielliptic genus 2 curves, Manuscripta Mathematica, 100 (1999), 519–533.

[22] Harris, Joe and Morrison, Ian, Moduli of Curves, Springer, (1998).

[23] Faltings, Gerd, Endlichkeitssätze für abelsche Varietäten über Zahlkörpern, Inventiones Mathematicae, 73 (1983), 349–366.

[24] Weil, André, Foundations of Algebraic Geometry, American Mathematical Society, 1946.

Some Results On Generalized Harmonic Maps

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Abstract.

In this work, we extend the definition of p-harmonic and p-biharmonic maps by introducing $p(\cdot)$-harmonic maps between two Riemannian manifolds, which are regarded as a generalization of harmonic maps, and p-harmonic maps, as well as stable $p(\cdot)$-harmonic maps as a generalization of stable harmonic maps and stable p-harmonic maps

References

[1] P. Baird, J. C. Wood, *Harmonic morphisms between Riemannain manifolds*, Clarendon Press, Oxford (2003).

[2] P. Baird, S. Gudmundsson, *p-Harmonic maps and minimal submanifolds*, Math. Ann. **294**, 611-624 (1992).

[3] B. Bojarski, T. Iwaniec, *p-Harmonic equation and quasiregular mappings*, Banach Center Publ. **19** (1), 25-38 (1987).

[4] L-F. Cheung, P-F. Leung, *Some results on stable p-harmonic maps*, Glasgow Math. J. **36**, 77-80 (1994).

[5] J. Eells, J. H. Sampson, *Harmonic mappings of Riemannian manifolds*, Amer. J. Math. **86**, 109-160 (1964).

[6] A. Fardoun, *On equivariant p-harmonic maps*, Ann.Inst. Henri. Poincare, **15**, 25-72 (1998).

[7] G. Y. Jiang, 2*-harmonic maps and their first and second variational formulas*, Chinese Ann. Math. Ser. A, **7** (4), 389-402, (1986).

[8] A. Mohammed Cherif, *On the p-harmonic and p-biharmonic maps*, J. Geom. 109:41, (2018).

[9] T. Nagano, M. Sumi, *Stability of p-harmonic maps*, Tokyo J. Math. **15** (2), 475-482 (1992).

[10] Y. Xin, *Geometry of harmonic maps*, Fudan University, (1996).

Bernoulli Polynomials as a Lens for Extending Schwindt's Congruence

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Abstract.

This work generalizes Schwindt's 1934 congruence by proving that for any integer $m \geq 1$ and prime $p \geq 7$ with $p \neq m+1$ and $p \neq 2m+1$, the identity

$$(1+2^{2m}) \sum_{r=1}^{\lfloor p/3 \rfloor} \frac{1}{r^{2m}} \equiv \sum_{r=1}^{\lfloor p/6 \rfloor} \frac{1}{r^{2m}} \pmod p$$

holds. The proof relies on properties of Bernoulli polynomials, p-integrality, and classical results of Lehmer. This identity extends a century-old result into a parametric family, revealing deeper symmetries in modular arithmetic.

Keywords

Schwindt congruence, Bernoulli numbers, Bernoulli polynomials, p-integers, von Staudt–Clausen theorem, Fermat's Last Theorem, Wieferich primes, harmonic sums modulo p, truncated power sums.

Main Result

The central achievement of this work is the following theorem.

Theorem 10.4 (Generalized Schwindt Congruence). *Let $m \in \mathbb{N}^*$ and $p \geq 7$ a prime number such that $p \neq m+1$ and $p \neq 2m+1$. Then*

$$(1+2^{2m}) \sum_{r=1}^{\lfloor p/3 \rfloor} \frac{1}{r^{2m}} \equiv \sum_{r=1}^{\lfloor p/6 \rfloor} \frac{1}{r^{2m}} \pmod p.$$

When $m=1$, the factor $1+2^2=5$ recovers the original congruence established by Paul Schwindt in 1934:

$$5 \sum_{r=1}^{\lfloor p/3 \rfloor} \frac{1}{r^2} \equiv \sum_{r=1}^{\lfloor p/6 \rfloor} \frac{1}{r^2} \pmod p, \quad p \geq 7.$$

Key Tools

The proof synthesizes several classical concepts:

- p-**integers**: The ring $\mathbb{Z}_{(p)} = \{a/b \in \mathbb{Q} : p \nmid b\}$ allows safe reduction of rational numbers modulo p.
- **Bernoulli numbers and polynomials**: Defined by

$$\frac{t}{e^t - 1} = \sum_{n=0}^{\infty} B_n \frac{t^n}{n!}, \quad \frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!},$$

- **von Staudt–Clausen theorem**: For even $n \geq 2$,

$$B_n + \sum_{\substack{\ell \text{ prime} \\ \ell - 1 | n}} \frac{1}{\ell} \in \mathbb{Z},$$

  which implies $B_n \in \mathbb{Z}_{(p)}$ if and only if $p - 1 \nmid n$.
- **Functional equation**: A key identity for rational arguments:

$$B_n\left(\tfrac{1}{6}\right) = \left((-1)^{n-1} + 2^{1-n}\right) B_n\left(\tfrac{1}{3}\right).$$

References

[1] F. Z. Bensaci and L. Khaldi, *A remark on Schwindt's congruence*, Mathematica Applicanda **52** (2024), no. 2, 327–334.

[2] P. Schwindt, *Über die Kongruenz* $\sum_{r=1}^{(p-1)/2} \frac{1}{r^2} \equiv 0 \pmod{p}$ *und verwandte Fragen*, Journal für die reine und angewandte Mathematik **171** (1934), 228–232.

[3] E. Lehmer, *On Congruences involving Bernoulli Numbers and the Quotients of Fermat and Wilson*, Annals of Mathematics **39** (1938), no. 2, 350–360.

On the generalized of p-biharmonic maps in Riemmanian manifolds

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Abstract.

In this paper , we broaden the definition of p-biharmonic and bi-p-harmonic maps between two Riemannian manifolds to explore this new concept of (p, q)-harmonic maps .

Keywords: p-biharmonic maps, bi-p-harmonic maps,(p, q)-harmonic maps.

References

[1] Baird, P., Wood, J. C.: *Harmonic morphisms between Riemannain manifolds*. Clarendon Press, Oxford (2003).

[2] Baird, P., Gudmundsson, S.: *p-Harmonic maps and minimal submanifolds*. Math. Ann. **294**, 611-624 (1992).

[3] Bojarski, B., Iwaniec, T.: *p-Harmonic equation and quasiregular mappings*. Banach Center Publ. **19** (1), 25-38 (1987).

[4] Eells, J., Sampson, J. H.: *Harmonic mappings of Riemannian manifolds*. Amer. J. Math. **86**, 109-160 (1964).

[5] Fardoun, A.: *On equivariant p-harmonic maps*. Ann. Inst. Henri. Poincare. **15**, 25-72 (1998).

[6] Jiang, G. Y.: 2-*harmonic maps and their first and second variational formulas*. Chinese Ann. Math. Ser. A. **7** (4), 389-402 (1986).

[7] Mohammed Cherif, A.: *On the p-harmonic and p-biharmonic maps*. J. Geom. **109** (41), (2018).

[8] Xin Y.: *Geometry of harmonic maps*. Fudan University (1996).

[9] Latti, F., Mohammed Cherif, A.: *On the generalized of -biharmonic and bi–harmonic maps*. arXiv:2603.06133.

[10] Tadj, M., Cherif, A.M., Latti, F.: *On the generalization of biharmonic hypersurfaces and biharmonic curves*. arXiv:2603.23692.

Curvature Invariants and Geometric Characterization of Hopf Bifurcation in Planar Dynamical Systems

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Abstract.

This work introduces a geometric framework for planar nonlinear dynamical systems, with emphasis on Hopf bifurcation and limit cycle formation [2],[4]. Instead of relying on linearization and spectral analysis, trajectories are interpreted as parametrized curves in the plane, allowing the use of curvature as a global geometric invariant. For a planar system

$$\dot{x} = f(x, y, \mu), \tag{1}$$
$$\dot{y} = g(x, y, \mu), \tag{2}$$

solutions are viewed as smooth curves $\gamma(t) = (x(t), y(t))$ with curvature $\kappa(t) = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$. A curvature-based invariant is defined by $I(\mu) = \langle \kappa(t) \rangle$, as the long-time average in the asymptotic regime. This provides a global geometric quantity associated with trajectory structure in phase space. The method is applied to the Hopf normal form, which undergoes a supercritical bifurcation at $\mu = 0$ (see [1]). For $\mu < 0$, trajectories converge to an equilibrium point, while for $\mu > 0$, they approach a periodic orbit. These regimes exhibit distinct geometric behavior: curvature decays in the stable case and remains persistent in the oscillatory regime. Consequently, the invariant $I(\mu)$ undergoes a qualitative transition at the bifurcation point, reflecting the emergence of limit cycles. Numerical simulations confirm this geometric signature. This approach provides a geometric characterization of Hopf bifurcation and highlights curvature as an invariant capturing global features of planar dynamical systems. Keywords:Hopf bifurcation, curvature invariant, planar dynamical systems, limit cycles, differential geometry

References

[1] S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Westview Press, 2015.

[2] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, 1983.

[3] E. Hopf, Abzweigung einer periodischen Lösung von einer stationären Lösung eines Differentialsystems, *Ber. Math.-Phys. Kl. Sächs. Akad. Wiss. Leipzig*, 94 (1942), 1–22.

Harmonic Maps and Harmonic Vector Fields on the Thurston Geometries F^4 and Nil^4

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Abstract. This work investigates harmonic maps and harmonic vector fields on two four-dimensional Thurston geometries: F^4 and Nil^4. Both spaces are Lie groups equipped with canonical left-invariant Riemannian metrics whose curvature tensors exhibit a rich structure. The geometry F^4 arises as the product manifold $\mathbb{R}^2 \times \mathbb{H}^2$ with full isometry group $\mathbb{R}^2 \rtimes SL_2(\mathbb{R})$, while Nil^4 is the four-dimensional nilpotent Lie group defined via a semidirect product $\mathbb{R}^3 \rtimes_{\mathcal{U}} \mathbb{R}$. Despite their structural differences, both geometries share a key feature: their Ricci curvature tensors satisfy a coercivity condition of the form $\mathrm{Ric} - \lambda g \geq 0$, which plays a central role throughout our analysis. A preliminary observation, common to both settings, establishes a natural link between Ricci soliton theory and harmonic analysis. We prove that the components of the Ricci soliton vector field ξ, expressed in the respective left-invariant frames, are harmonic functions with respect to the Laplace-Beltrami operator. This result reveals an intrinsic compatibility between the solitonic structure of each geometry and its harmonic function theory, motivating a systematic investigation of harmonic maps and vector fields in these settings. The first main results concern the non-existence of non-trivial harmonic maps into these geometries. Applying a Bochner-type argument, which guarantees the triviality of harmonic maps whenever the target satisfies $\mathrm{Ric} - \lambda g \geq 0$, we establish a Liouville-type theorem in each case. For F^4, the coercivity condition reads $\mathrm{Ric}(X, X) - \lambda g(X, X) = 6(X_1^2 + X_2^2) \geq 0$ for all tangent vectors X. For Nil^4, the corresponding inequality is $\mathrm{Ric}(X, X) - \lambda g(X, X) = 2X_1^2 + \frac{3}{2}X_2^2 + X_3^2 + \frac{1}{2}X_4^2 \geq 0$. In both cases, we conclude that any harmonic map from a compact orientable Riemannian manifold without boundary into either geometry must be constant, confirming the rigidity imposed by these spaces on the class of admissible harmonic maps from compact domains. We then characterize harmonic vector fields on both geometries from two complementary perspectives. Regarding harmonic sections, we seek vector fields that are critical points of the vertical energy functional. On F^4, considering components depending on coordinates s and t, the harmonicity conditions reduce to a coupled system of partial differential equations. Several families of explicit solutions are classified: e.g., a field of the form $X_1(s,t)\partial_x$ is harmonic if and only if $X_1 = c_1 + c_2 t^2$, while the components along ∂_s and ∂_t involve power-type solutions $c_1 t^{\frac{3}{2}+\frac{\sqrt{7}}{2}} + c_2 t^{\frac{3}{2}-\frac{\sqrt{7}}{2}}$. On Nil^4, seeking components depending only on t, the harmonicity equations form a coupled PDE system. The components along e_1 and e_3 admit only the trivial harmonic section, while the component along e_4 yields the non-trivial solution $X_4(t) = c_1 e^t + c_2 e^{-t}$. These results provide a complete and explicit description of

the harmonic section landscape on both geometries. In the second approach, vector fields are studied as harmonic maps into the tangent bundle equipped with the Sasaki metric, a strictly stronger condition that couples the curvature of the manifold with the covariant derivatives of the field. On F^4, a complete analysis of the resulting nonlinear system reveals that the unique solution is the trivial vector field $X \equiv 0$, exhibiting sharp geometric rigidity. On Nil^4, the situation is richer: the system admits two non-trivial families of harmonic maps into the tangent bundle, namely $X = (c_1 e^t + c_2 e^{-t})e_4$, and $X = X_1 e_1 + X_1 e_3$ with $X_1 = c_1 e^{\frac{t}{\sqrt{2}}} + c_2 e^{\frac{-t}{\sqrt{2}}}$. This contrast highlights how the nilpotent structure of Nil^4 permits a wider class of harmonic objects than the hyperbolic-type geometry of F^4. Taken together, these results provide a coherent comparative picture of harmonic theory on two distinct four-dimensional Thurston geometries. Both F^4 and Nil^4 exhibit strong rigidity for harmonic maps from compact domains, yet display markedly different behaviors for harmonic vector fields and harmonic maps into the tangent bundle. The methods developed here, combining Lie group techniques, Bochner-type formulas, and the geometry of the Sasaki metric, are expected to extend to other Thurston geometries, enriching our understanding of geometric analysis on left-invariant structures.

References

[1] Baird, P.; Wood, J.C. *Harmonic Morphisms Between Riemannian Manifolds.* Clarendon Press: Oxford, UK, 2003.

[2] Boukhari, H.; Okbani, H.; Mohammed Cherif, A. *On Ricci Solitons and Harmonic Vector Fields in the Thurston Geometry F^4*. arXiv:2603.08969 .

[3] Calvaruso, G.; De Leo, B. *Curvature properties of four-dimensional generalized symmetric spaces.* J. Geom. 2008, 90, 30–46.

[4] Cherif, A.M. *On the nonexistence of harmonic and bi-harmonic maps.* Bull. Math. Soc. Sci. Math. Roumanie 2025, 68(116), No. 2, 173–184.

[5] Eells, J.; Sampson, J.H. *Harmonic mappings of Riemannian manifolds.* Am. J. Math. 1964, 86, 109–160.

[6] Filipkiewicz, R. *Four Dimensional Geometries.* Ph.D. Thesis, University of Warwick, Coventry, UK, 1983.

[7] Gil-Medrano, O. *Relationship between volume and energy of vector fields.* Differ. Geom. Appl. 2001, 15, 137–152.

[8] Gudmundsson, S.; Kappos, E. *On the geometry of tangent bundles.* Expo. Math. 2002, 20, 1–41.

[9] Li, Y.; Cherif, A.M.; Xie, Y. *Characterization of Ricci solitons and harmonic vector fields on the Lie group Nil^4*. Mathematics 2025, 13, 1155.

Algebraic Structures of Hulls in Linear Codes over Finite Fields

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Abstract.

We study the hull dimension of linear codes over finite fields. Depending on the chosen inner product, we show that any code can be transformed via monomial equivalence into a code whose hull dimension is reduced to zero. We also explore pure LCD codes and extend our results to relative hulls.

Key words: Hull, Relative Hull, LCD code, pure LCD codes, Monomial equivalence.

References

[1] TANG, Claude Carlet1 Sihem Mesnager 2 Chunming et PELLIKAAN, Yanfeng Qi4 Ruud. Linear codes over Fg are equivalent to LCD codes for g¿ 3. IEEE Transactions on Information Theory, 2018, vol. 64, no 4, p. 3010-3017.

[2] Massey, J. L. (1992). Linear codes with complementary duals. Discrete Mathematics, 106, 337-342.

[3] YOUCEF, Maouche. Linear codes with arbitrary dimensional hull and pure LCD code. arXiv preprint arXiv:2306.00285, 2023.

[4] ANDERSON, Sarah E., CAMPS-MORENO, Eduardo, LÓPEZ, Hiram H., et al. Relative hulls and quantum codes. IEEE Transactions on Information Theory, 2024.

Codazzi hypersurfaces in $\mathrm{Nil}^3 \times \mathbb{R}$

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Abstract.

We give a classification of hypersurfaces whose second fundamental form is a Codazzi tensor, in particular totally geodesic hypersurfaces, hypersurfaces with parallel second fundamental form and totally umbilical hypersurfaces in $\mathrm{Nil}^3 \times \mathbb{R}$.

The Geodesic Structure of Takano Gaussian Space

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Abstract.

Model and Information Geometry Framework
We consider the isotropic multivariate Gaussian model

$$X \sim \mathcal{N}(m, \sigma^2 I_n), \quad m \in \mathbb{R}^n,\ \sigma > 0.$$

The log-likelihood function is given by

$$\ell(x, m, \sigma) = -\frac{n}{2}\log(2\pi) - n\log\sigma - \frac{1}{2\sigma^2}\|x - m\|^2.$$

Metric and Differential Structure
The partial derivatives of the log-likelihood are

$$\partial_{m_i}\ell = \frac{x_i - m_i}{\sigma^2}, \quad \partial_\sigma \ell = -\frac{n}{\sigma} + \frac{\|x-m\|^2}{\sigma^3}.$$

The Fisher information metric is defined by

$$g_{ij} = \mathbb{E}[\partial_i \ell \partial_j \ell],$$

and is given by

$$g_{m_i m_j} = \frac{1}{\sigma^2}\delta_{ij}, \quad g_{m_i\sigma} = 0, \quad g_{\sigma\sigma} = \frac{2n}{\sigma^2}.$$

Tensor and α-Connection
The cubic tensor

$$T_{ijk} = \mathbb{E}[\partial_i \ell \partial_j \ell \partial_k \ell]$$

has the non-zero components

$$T_{ij\sigma} = \frac{2}{\sigma^3}\delta_{ij}, \quad T_{\sigma\sigma\sigma} = \frac{8n}{\sigma^3}.$$

The Levi-Civita connection components are

$$\Gamma^{(0)}_{ij\sigma} = -\frac{1}{\sigma^3}\delta_{ij}, \quad \Gamma^{(0)}_{\sigma\sigma\sigma} = -\frac{2n}{\sigma^3}.$$

The α-connection is defined by

$$\Gamma^{(\alpha)}_{ijk} = \Gamma^{(0)}_{ijk} - \frac{\alpha}{2} T_{ijk},$$

yielding

$$\Gamma^{(\alpha)}_{ij\sigma} = -\frac{1+\alpha}{\sigma^3}\delta_{ij}, \quad \Gamma^{(\alpha)}_{\sigma\sigma\sigma} = -(1+2\alpha)\frac{2n}{\sigma^3}.$$

Derivatives in Takano Gaussian Space

Using the inverse Fisher metric

$$g^{\sigma\sigma} = \frac{\sigma^2}{2n}, \quad g^{ij} = \sigma^2 \delta^{ij},$$

the covariant derivatives are obtained as

$$\nabla^{(\alpha)}_{\partial_i} \partial_j = \frac{1-\alpha}{2n\sigma} \delta_{ij} \partial_\sigma,$$

$$\nabla^{(\alpha)}_{\partial_i} \partial_\sigma = -\frac{1+\alpha}{\sigma} \partial_i,$$

$$\nabla^{(\alpha)}_{\partial_\sigma} \partial_\sigma = -\frac{1+2\alpha}{\sigma} \partial_\sigma.$$

References

[1] S.-i. Amari and H. Nagaoka, *Methods of Information Geometry*, American Mathematical Society, 2000.

[2] S.-i. Amari, *Information Geometry and Its Applications*, Springer, 2016.

[3] N. Ay, J. Jost, H. V. Lê, and L. Schwachhöfer, *Information Geometry*, Springer, 2017.

New types metric deformation of harmonic identity maps

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Abstract

An identity map $(M, g) \longrightarrow (M, g)$ is a harmonic from a Riemannian manifold (M, g) onto itself. In this paper, we study the harmonicity of identity maps $(M, g) \longrightarrow (M, g - df \otimes df)$ and $(M, g - df \otimes df) \longrightarrow (M, g)$ where f is a smooth function with gradient norm < 1 on (M, g). We construct new examples of identity harmonic maps. We define a symmetric tensor field on M whose properties are related to the harmonicity of these identity maps.

References

[1] P. Baird, J. C. Wood, *Harmonic morphisms between Riemannain manifolds*, Clarendon Press, Oxford (2003).

[2] A. Benkartab, A. Mohammed Cherif, *New methods of construction for biharmonic maps*, Kyungpook Math. J. **59**(2019), 135–147.

[3] A. Benkartab, A. Mohammed Cherif, *Deformations of Metrics and Biharmonic Maps*, Communications in Mathematics, **28** (2020), 263–275.

[4] J. Eells, L. Lemaire,*A report on harmonic maps*, Bull. London Math. Soc. **16** (1978), 1–68.

[5] J. Eells, L. Lemaire, *Another report on harmonic maps*, Bull. London Math. Soc. **20** (1988), 385–524.

[6] J. Eells and J. H. Sampson, *Harmonic mappings of Riemannian manifolds*, Amer. J. Math. **86** (1964), 109–160.

[7] B. Merdji, A. Mohammed Cherif, New types of metrics deformations and applications to p-biharmonic maps, Journal of the Indian Math. Soc., **90**, (3-4) (2023), 387–400.

[8] C. Udrişte, *Convex Functions and Optimization Methods on Riemannian Manifolds*, Mathematics and Its Applications, Kluwer Academic Publishers: Dordrecht, The Netherlands, 1994.

[9] T. Sakai, *Riemannian geometry*, Shokabo, Tokyo (1992). (in Japanese).

On groups whose proper subgroups of infinite rank are finite-by-locally nilpotent

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Abstract.

A group G is said to be of finite rank r if every finitely generated subgroup of G can be generated by at most r elements, and r is the least positive integer with a such property. If there is no such r, then the group G is said to be of infinite rank. In recent years, many authors studied the structure of locally (soluble-by-finite) groups G of infinite rank in which every proper subgroup of infinite rank belongs to a given class $\mathfrak{Y}$ and they proved that all proper subgroups of G belong to $\mathfrak{Y}$, sometimes the group G itself belongs to $\mathfrak{Y}$ (see for instance, [2, 3, 4]). In particular, it is proved in [4, Theorem B'], that an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are locally nilpotent is itself locally nilpotent, where $\mathfrak{X}$ is the class introduced in [1] as the class obtained by taking the closure of the class of periodic locally graded groups by the closure operations $\acute{\boldsymbol{P}}$, $\grave{\boldsymbol{P}}$ and $\boldsymbol{L}$. Recall that a group is said to be locally graded if every non-trivial finitely generated subgroup contains a proper subgroup of finite index. The class $\mathfrak{X}$ is a subclass of the class of locally graded groups that contains all locally (soluble-by-finite) groups and in [1], it is proved that an $\mathfrak{X}$-group of finite rank is almost locally soluble. Recently, in [3], it is proved that an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are finite-by-hypercentral (respectively, hypercentral-by-finite) has all its proper subgroups finite-by-hypercentral (respectively, hypercentral-by-finite). In the present work, we consider this problem for the class of finite-by-locally nilpotent and we prove that an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are finite-by-locally nilpotent has all its proper subgroups finite-by-locally nilpotent.

Note that the consideration of the famous example of Heineken and Mohamed [4], shows that a group that satisfies the hypothesises of our result is not, in general, finite-by-locally nilpotent.

References

[1] N. S. Černikov, *A theorem on groups of finite special rank.* J. Ukrain. Math. 42, 855-861, 1990.

[2] M. De Falco, F. De Giovanni, C. Musella and N. Trabelsi, *Groups with restrictions on subgroups of infinite rank.* Rev. Mat. Iberoam, 30 (2), 537-550, 2014.

[3] A. Dilmi and N. Trabelsi, *Groups whose proper subgroups of infinite rank are finite-by-hypercentral or hypercentral-by-finite*, Acta Math. Hung. **167**: 492-500, 2022.

[4] M. R. Dixon, M. J. Evans and H. Smith, *Locally (soluble-by-finite) groups with all proper non-nilpotent subgroups of finite rank*. J. Pure Appl. Algebra, **135**: 33-43, 1999.

Application of Deep Machine Learning Methods to Mathematical Physics of nonlinear system to track desired geometric trajectory

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Abstract.

The modeling and control of complex nonlinear physical systems [1]—including unmanned aerial vehicles [2] and robotic manipulators—remain fundamental challenges in modern mathematical physics, primarily due to the presence of strong nonlinear couplings, high-dimensional state spaces, and intrinsic uncertainties. Classical control methodologies [3], while theoretically well-established, often rely on restrictive assumptions (e.g., precise model knowledge, weak nonlinearities) that limit their effectiveness in realistic and highly uncertain environments.

Mathematical Core. A broad class of nonlinear control systems can be formulated on a smooth manifold M as the control-affine system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x), u_i, \qquad x \in M, \tag{3}$$

where $f, g_1, \ldots, g_m$ are smooth vector fields. This formulation naturally captures the geometric structure of the dynamics and provides a foundation for coordinate-free analysis using tools from differential geometry and Lie theory.

In parallel, the rapid development of deep machine learning techniques [4] has opened new perspectives for the analysis of experimental data and the modeling of complex physical processes. These approaches enable the extraction of latent structures and nonlinear relationships from large-scale datasets, thereby complementing classical first-principles modeling. In the context of mathematical physics [5], such data-driven methodologies facilitate the approximation of unknown dynamics, enhance predictive capabilities, and support the validation of theoretical models against empirical observations.

More specifically, the analysis of experimental data provides critical insight into system behavior over time, enabling the identification of dominant patterns and the formulation of physically consistent hypotheses. This data-centric perspective plays a crucial role in refining mathematical models, improving parameter estimation, and enhancing the overall fidelity of system representations.

Motivated by these considerations, this work proposes a unified framework that integrates differential geometric methods with deep learning techniques to address trajectory tracking problems for nonlinear dynamical systems. The proposed approach leverages the intrinsic geometric structure of the system by formulating tracking objectives on appropriate manifolds, while employing data-driven approximators to capture unknown or unmodeled nonlinearities. This synergy between geometric control and learning enhances both modeling accuracy and control performance.

Finally, a critical assessment of the advantages and limitations of deep learning methods is conducted, providing insight into their effectiveness, robustness, and potential challenges when deployed in practical applications. This analysis highlights the importance of structure-preserving learning strategies to ensure consistency with the underlying physical and geometric principles.

References

[1] C. M. Cheng, Z. K. Peng, W. M. Zhang, and G. Meng, "Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review," *Mechanical Systems and Signal Processing*, vol. 87, pp. 340–364, 2017.

[2] S. Hwang, M. Cho, and I. Hwang, "LMI-Driven Tracking Control of Fuzzy Nonlinear Cyber-Physical Systems: Application to Quadrotor UAVs in Urban-Like Environment," in *AIAA SCITECH 2026 Forum*, p. 1584, 2026.

[3] H. K. Khalil, *Nonlinear Systems*, 3rd ed., Prentice Hall, 2002.

[4] P. P. Shinde and S. Shah, "A review of machine learning and deep learning applications," in *Proc. 2018 Fourth International Conference on Computing, Communication, Control and Automation (ICCUBEA)*, pp. 1–6, 2018.

[5] A. Sanchez-Gonzalez, J. Godwin, T. Pfaff, R. Ying, J. Leskovec, and P. Battaglia, "Learning to simulate complex physics with graph networks," in *Proc. International Conference on Machine Learning (ICML)*, pp. 8459–8468, 2020.

Analyzing Dynamics of the Modified Gardner Equation: Lie Symmetry, Bifurcation, Chaos, and Multistability Analysis

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Abstract.

The present work investigates the dynamical behavior of the Modified Gardner equation, a significant nonlinear model arising in the context of wave propagation and complex systems. Lie symmetry analysis is employed to construct symmetry reductions of the governing equation, facilitating the derivation of invariant solutions and simplifying its analytical structure. The reduced dynamical system is further examined through bifurcation analysis and phase portrait techniques to characterize its qualitative behavior. Bifurcation phenomena are observed at critical parameter values of the system, where the application of perturbation induces a transition leading to chaotic dynamics. The onset of chaos is further confirmed through phase space analysis, time series evolution, multistability behavior, and Lyapunov exponent calculations, which collectively demonstrate the sensitive dependence on initial conditions and the presence of irregular motion. In addition, the modified Sardar Sub-equation (MSSE) method is utilized to construct a variety of exact soliton solutions, including bright, dark, singular, periodic, combo, bright–dark, rational, and mixed trigonometric wave structures. The obtained results reveal both consistency with existing literature and the emergence of novel wave solutions, highlighting the richness of the model. The study provides new insights into the interplay between symmetry, bifurcation, and chaotic behavior in nonlinear wave systems, contributing to the understanding of complex dynamics in applied mathematics and mathematical physics.

References

[1] B. S. T. Alkahtan, Exploration of Lie Symmetry, Bifurcation, Chaos and Exact Solution of the Geophysical KdV Equation, *International Journal of Theoretical Physics*, vol. 64, p. 79, 2025. doi: 10.1007/s10773-025-05934-6.

[2] A. Jhangeer, A. Hussain, M. Junaid-U-Rehman, D. Baleanu, and M. B. Riaz, Quasi-periodic, chaotic and travelling wave structures of modified Gardner equation, *Chaos, Solitons and Fractals*, vol. 143, p. 110578, 2021. doi: 10.1016/j.chaos.2020.110578.

[3] N. K. Das, Y. Chettri, A. Das, and A. Saha, Periodic, multi-periodic, quasiperiodic and shock ocean waves through the geophysical-Burgers equation, *Physics Letters A*, vol. 570, p. 131236, 2026. doi: 10.1016/j.physleta.2026.131236.

On $D(-1)$-Fibonacci triple

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Abstract.

Let $k \geq 0$ be an integer and F_k be the k^{th} Fibonacci number. In this talk, we purpose to prove that it does not exist any $D(-1)$-Fibonacci triple.

References

[1] C. Fuchs, F. Luca and L. Szalay, *Diophantine triples with values in binary recurrences*, Ann. Sc. Norm. Super. Pisa Cl. Sc. **5.7)** (2008), 579–608.

[2] F. Luca and L. Szalay, *Fibonacci Diophantine triples*, Glasnik Matematički. Vol. **43** (2008), 253 264

On Locally Graded Minimal Non-(Hypercentral-by-Finite) Groups

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Abstract.

In this note, we give a characterization of locally graded minimal non-(hypercentral-by-finite) groups in terms of minimal non-hypercentral groups and minimal non-(abelian-by-finite) groups. More precisely, we prove that if G is a locally graded minimal non-(hypercentral-by-finite) group, then G is a countable torsion group of infinite rank such that G' is hypercentral and G/G' is a quasicyclic p-group. Moreover, either G is a locally nilpotent minimal non-hypercentral group, or G' is a q-group for some prime $q \neq p$ and G/G'' is a non-locally nilpotent minimal non-(abelian-by-finite) group.

Keywords: Locally graded groups, hypercentral-by-finite groups, abelian-by-finite groups, minimal non-Ω groups.

References

[1] A. Arikan and N. Trabelsi, On minimal non-Baer groups, *Communications in Algebra*, 39 (2011), 2489–2497.

[2] A. Arikan and N. Trabelsi, On groups whose proper subgroups are Chernikov-by-Baer, *J. Algebra Appl.*, 12 (2013), 1–15.

[3] S. Azra and N. Trabelsi, On locally graded minimal non-(finite-by-hypercentral) groups, *Bull. Malaysian Math. Sci. Soc.*, 44 (2021), 1–8.

[4] A. Badis and N. Trabelsi, Soluble minimal non-(finite-by-Baer) groups, *Ricerche Mat.*, 59 (2010), 129–135.

[5] B. Bruno, On groups with abelian-by-finite proper subgroups, *Boll. U.M.I.*, 3-B (1984), 797–807.

[6] B. Bruno and R. E. Phillips, Groups with nilpotent-by-finite proper subgroups, *Arch. Math.*, 65 (1995), 369–374.

[7] F. De Giovanni and M. Trombetti, Infinite minimal non-hypercyclic groups, *J. Algebra Appl.*, 14 (2015).

[8] D. J. S. Robinson, *Finiteness Conditions and Generalized Soluble Groups*, Springer, 1972.

[9] M. Xu, Groups whose proper subgroups are finite-by-nilpotent, *Arch. Math.*, 66 (2009), 353–359.

[10] M. Xu, Groups whose proper subgroups are Baer groups, *Acta Mathematica Sinica*, 12 (1996), 10–17.

Eigenvalue criteria for existence and nonexistence of positive solutions for fractional differential equations of order $2 < \alpha < 3$ on the half-line

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Abstract.

This article concerns nonexistence and existence of positive solutions to the fractional differential equation

$$\begin{cases} D^{\alpha}u(t) + f(t, u(t)) = 0, \ 0 \leq t < \infty \\ u(0) = D^{\alpha-2}u(0) = \lim_{t\to\infty} D^{\alpha-1}u(t) = 0 \end{cases}$$

where $\alpha \in (2, 3)$, D^{α} is the standard Riemann-Liouville derivative and $f : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function. The main results obtained here are under eigenvalue criteria.

Key Words: Fractional differential equation; Positive solutions; Fixed point theory in cones.

Introduction

Because fractional differential equations are considered as alternative models to nonlinear differential equations, study of existence of positive solutions to boundary value problems associated with fractional differential equations has become a very important area of applied mathematics over the last few decades. Such a subject has been discussed in many recent papers; see, for example [5], [6], [7], and references therein.

We are concerned in this paper with nonexistence and existence of positive solutions to the fractional boundary value problem (fbvp for short),

$$\begin{cases} D^{\alpha}u(t) + f(t, u(t)) = 0, \ 0 \leq t < \infty \\ u(0) = D^{\alpha-2}u(0) = \lim_{t\to\infty} D^{\alpha-1}u(t) = 0 \end{cases} \tag{4}$$

where $\alpha \in (2, 3)$, D^{α} is the standard Riemann-Liouville derivative and $f : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function.

Motivated by the works in [1], [2], [3], [4], we want to establish nonexistence and existence results to the fbvp (4) under eigenvalue criteria.

Set

$$\mathcal{Q}_\alpha = \left\{ q \in C\left(\mathbb{R}^+, \mathbb{R}\right) : q(s) > 0 \text{ a.e. } s > 0 \text{ and } \int_0^{+\infty} q(s)\,(1+s)^{\alpha-1}\,ds < \infty \right\}.$$

A continuous function $g : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is said to be $\mathcal{Q}_\alpha$-Carathéodory if for all $r > 0$ there is $\psi_r \in \mathcal{Q}_\alpha$ such that

$$\left| g\left(t, (1+t)^{\alpha-1} u\right) \right| \leq \psi_r(t)\,(1+t)^{\alpha-1} \text{ for all } t \in \mathbb{R}^+ \text{ and } u \in [-r, r].$$

A continuous function $g : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ such that

$$\left| (1+t)^{1-\alpha} g(t, u) \right| \leq a(t) + b(t)\,|u|^\rho \text{ for all } t, u \in \mathbb{R}^+$$

where $\rho \in (0, +\infty)$ and $a, \widetilde{b} \in \mathcal{Q}_\alpha$ with $\widetilde{b}(s) = (1+s)^{(\alpha-1)(\rho-1)}\, b(s)$, is a typical $\mathcal{Q}_\alpha$-Carathéodory function.

Then the fbvp (4) admits a positive solution.

Corollary 10.5. *Assume that $m \in \mathcal{Q}_\alpha$ where $m(t) = p(t)\,(1+t)^{(\alpha-1)(\rho-1)}$. Then for all $\rho \in (0,1) \cup (1, +\infty)$ the fbvp*

$$\begin{cases} D^\alpha u(t) + p(t) u^\rho = 0, \ t > 0 \\ u(0) = D^{\alpha-2} u(0) = 0, \ \lim_{t\to\infty} D^{\alpha-1} u(t) = 0, \end{cases} \tag{5}$$

where $\rho > 0$ and $p \in C(\mathbb{R}^+, \mathbb{R}^+)$, admits a positive solution.

References

[1] A. Benmezaï, Positive solutions for a second order two point boundary value problem, *Commun. Appl. Anal.* **14** (2010), no. 2, 177-190.

[2] A. Benmezaï, W. Esserhane and J. Henderson, *Existence of positive solutions for singular second order boundary value problems under eigenvalue criteria*, *Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal.* **20** (2013), 709-725.

[3] A. Benmezaï, J. R. Graef and L. Kong, *Positive solutions to a two point singular boundary value problem*, *Differ. Equ. Appl.* **3** (2011), no. 3, 347-373.

[4] A. Benmezaï, John R. Graef and L. Kong, *Positive solutions for the abstract Hammerstein equations and applications*, *Commun. Math. Anal.* **16** (2014), no. 1, 47-65.

[5] A. Cabada and G. Wang, *Positive solutions of nonlinear fractional differential equations with integral boundary value conditions*, *J. Math. Anal. Appl.* **389** (2012), 403-411.

[6] J. Caballero, I. Cabrera, and K. Sadarangani, *Positive solutions of nonlinear fractional differential equations with integral boundary value conditions*, *Abstr. Appl. Anal.* **2012**, Article ID 303545, 11 pages.

[7] M. Feng, X. Zhang and W. Ge, *New existence results for higher-order nonlinear fractional differential equations with integral boundary conditions*, *Bound. Value Probl.* **2011**, Article ID 720702, 20 pages.

On Hom-Lie algebra structures and their non-associative generalizations

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Abstract.

This communication explores recent developments within the theory of Hom-Lie algebras, a class of non-associative algebras where the classical Jacobi identity is twisted by an algebra morphism. Initially motivated by deformations of the Witt and Virasoro algebras in mathematical physics, this structure provides a robust framework for studying discrete symmetries. We present new constructions based on derivations and examine their classification properties for low dimensions. Emphasis is placed on characterizing adjoint-type representations and studying the associated cohomology. Finally, we discuss the extension of these results to Hom-Lie superalgebras, opening perspectives on the quantization of non-commutative dynamical systems.

References

[1] Kitouni, A., Silvestrov, S. (2024). "On Properties and Classification of a Class of 4-Dimensional 3-Hom-Lie Algebras with a Nilpotent Twisting Map". Axioms. Cet article explore la classification en petite dimension (4D) avec un focus sur les applications de torsion nilpotentes.

[2] Ben Jmaa, T., Makhlouf, A., Saadaoui, N. (2022). "Current Hom-Lie algebras". Journal of Algebra and Its Applications (ou via ResearchGate). Une étude sur les produits tensoriels et la classification des structures de Hom-Lie de dim 4.

[3] Mabrouk, S., et al. (2023). "Fuzzy Hom–Lie Ideals of Hom–Lie Algebras". MDPI Mathematics. Une extension moderne vers la logique floue appliquée aux structures de Hom-Lie, traitant des sous-algèbres et des idéaux.

Enhancing Cryptography with Integral Equations: A Position-Dependent Affine Cipher Based on Modular Arithmetic

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Abstract.

Modern cryptography relies on algebraic structures such as cyclic groups for public-key systems and Fredholm integral equations for emerging paradigms. This paper proposes a **generalized affine cipher** with **dynamic keys** generated from solutions of integral equations, enhancing both classical substitution ciphers and optical cryptography applications.

Introduction and Algebraic Background

The classical affine cipher encrypts a plaintext symbol x as $y \equiv \alpha x + \beta \pmod{m}$, where α is coprime to m and β is the shift. Decryption requires the modular inverse $\alpha^{-1} \pmod{m}$. This work generalizes the affine cipher by making the keys (α, β) **dynamic**, derived from the solution of a Fredholm integral equation of the second kind:

$$u(x) = v(x) + \int_a^b K(x,s)\, u(s)\, ds,$$

where the kernel $K(x,s)$ or its spectral representation generates time-varying or data-dependent parameters $\alpha(t)$ and $\beta(t)$.

Security is strengthened by linking the key stream to the ill-posed inverse problem, requiring regularization techniques for stable decryption. This approach combines the simplicity of affine transformations with the analytic depth of integral operators.

Cyclic Groups in Affine and Public-Key Cryptography

Affine ciphers operate over the ring $\mathbb{Z}/m\mathbb{Z}$. When $m = p$ (prime), the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^*$ is cyclic. Dynamic keys can exploit generators of cyclic subgroups to produce pseudo-random sequences for α and β, resisting frequency analysis better than static affine ciphers.

This generalization connects naturally to Diffie-Hellman and elliptic curve cryptography, where scalar multiplication in cyclic groups $E(\mathbb{F}_p)$ (points on Weierstrass curves) can seed the integral kernel parameters, creating hybrid algebraic-analytic schemes.

Integral Equation-Based Dynamic Key Generation

The core innovation lies in deriving dynamic keys from Fredholm integral equations. The plaintext (or a seed function) $u(s)$ is transformed via a secret symmetric kernel to produce a spectral coefficient set. These coefficients are then mapped algebraically to affine parameters:

$$\alpha_n = u(\lambda_n), \quad \beta_n = v(\mu_n),$$

where λ_n, μ_n arise from the discrete differential spectrum or Laplace transform of the integral solution.

For optical cryptography, the kernel can represent physical parameters (e.g., phase masks or refractive index distributions), allowing implementation via optical Fourier or fractional Fourier transforms. Decryption solves the inverse integral problem, stabilized by Tikhonov regularization or differential transformation methods.

This yields a cipher resistant to known-plaintext attacks due to the continuous-to-discrete mapping and the ill-posed nature of the Fredholm operator.

Conclusion

The proposed generalized affine cipher with dynamic keys derived from integral equations bridges classical modular arithmetic, cyclic group theory, and functional analysis. It provides a flexible framework for both classical and optical cryptographic applications, with promising resistance against classical and quantum threats. Future work will focus on rigorous security proofs and hardware implementations in optical systems.

References

[1] Bronshpak, G. et al. (2014). Cryptography of a new generation: integral equations as an alternative of algebraic methodology. *Prikladnaya Elektronika*.

[2] Hryshchuk, O. (2021). Spectral model of the encryption key based on differential transformations. CEUR-WS.

[3] Al-Azzani, A.M. et al. (2021). A new cryptography scheme based on Laplace transform. *Int. J. Advanced Trends in Computer Science and Engineering*.

[4] Washington, L.C. (2008). *Elliptic Curves: Number Theory and Cryptography*. Chapman & Hall/CRC.

[5] Katz, J. & Lindell, Y. (2020). *Introduction to Modern Cryptography*. CRC Press.

[6] Mani, A. (2026). A numerical method for solving state-dependent Chandrasekhar integra (in preparation).

On the initial value problem of functional differential equations

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Abstract.

In this work we provide sufficient conditions for the existence as well as the uniqueness of solution of general nonlinear functional differential equations. Our results will be obtained using suitable fixed point theorems. We give also some remarks about uniqueness theorem due to Dhage.

References

[1] A. Bouraada and M. Helal, *On the initial value problem of functional differential equations*, MESA Journal, Vol 14(1), (2023), 175-182.

[2] F.E Browder, Nonlinear operators and nonlinear equations of evolution in Banach spaces, *Porc.Symp.Pure Math.Amer.Math.Soc.Providence,Rhde Island*, (1976).

[3] B.C. Dhage, Existence theory for functional initial value problems of ordinary differential equations, *EJQTDE*, (2004), N0 15.

[4] J. Dugundji and A. Granas, Fixed point theory, *Monographie Math*, Warsaw, 1982.

Geometric Structure of the Friedmann–Lemaître–Robertson–Walker metric and Its Role in Modern Cosmology

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Abstract.

The Friedmann–Lemaître–Robertson–Walker metric provides the standard geometric model for describing the large-scale structure of the universe. Assuming homogeneity and isotropy, this metric reduces the complexity of spacetime to a form governed by a single time-dependent function, known as the scale factor.

In this talk, we present the construction of the FLRW metric from a geometric point of view and outline the computation of its main curvature quantities. We then show how the Einstein Field Equations lead naturally to the Friedmann Equations, which describe the dynamics of cosmic expansion.

The goal is to highlight how simple geometric assumptions lead to one of the most important models in modern cosmology.

References

[1] A. Liddle, An Introduction to Modern Cosmology, 2003

[2] B. O'Neill, Semi-Riemannian Geometry, 1983

[3] R. Wald, General Relativity, 1984

[4] S. Carroll, Spacetime and Geometry, 2004

Existence and Multiplicity of Positive Solutions for Fourth-Order Boundary Value Problems

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Abstract.

This work is part of the qualitative study of nonlinear higher-order differential equations, with a particular focus on boundary value problems of order greater than or equal to two. Such problems naturally arise in various models from mechanics, beam theory, and other areas of applied sciences. We are particularly concerned with the analysis of the solvability of these problems, with special emphasis on fundamental properties of positive solutions such as existence, multiplicity, uniqueness, and localization. The study of these properties plays a crucial role in understanding the behavior of systems governed by such equations. In this context, the main objective is to establish sufficient conditions ensuring the existence of positive solutions for a class of fourth-order boundary value problems. To achieve this, the considered differential problem is reformulated as an equivalent integral equation by means of the associated Green's function, reducing it to a fixed point problem within an appropriate functional framework. More precisely, we work in a Banach space equipped with a positive cone and associate with the problem an integral operator whose continuity and compactness properties are analyzed. The adopted approach relies mainly on fixed point theory in cones, particularly on a variant of the Birkhoff–Kellogg theorem and its subsequent developments. This topological method makes it possible to effectively handle positivity constraints and to obtain existence results under relatively general assumptions, while avoiding some of the restrictive conditions required by classical approaches. The obtained results are consistent with existing works on higher-order differential equations and contribute to enriching the available analytical methods in this field. Moreover, the developed framework can be adapted to other types of nonlinear boundary value problems, opening the way for further research.

References

[1] Han G., Xu Z., *Multiple solutions of some nonlinear fourth-order beam equations*, Nonlinear Anal., **68** (2008), no. 12, 3646-3656.

[2] Liu B., *Positive solutions of fourth-order two point boundary value problems*, Appl. Math. Comput., **148** (2004), no. 2, 407-420.

[3] Pang C., Dong W., Wei Z., *Multiple solutions for fourth-order boundary value problem*, J. Math. Anal. Appl., **314** (2006), no. 2, 464-476.

On Products of Terms from Two Binary Recurrence Sequences

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Abstract.

Exponential Diophantine equations, in which integer solutions are sought and at least one variable appears in the exponent, occupy a central place in number theory. Their inherent difficulty arises from their nonlinear structure and the rapid growth of exponential functions. Beyond their theoretical significance, such equations also emerge in various applied fields, including cryptography, coding theory, and discrete mathematical modeling.

In this work, we investigate the parametric exponential Diophantine equation

$$(a^k - 1)(b^l - 1) = c^m - 1.$$

This class of equations highlights intricate relationships between exponential expressions and structures related to binary recurrence sequences, providing a broader framework for understanding problems in algebraic and computational number theory.

We establish results toward the resolution of this equation using tools from transcendence theory and Diophantine approximation. In particular, our approach relies on lower bounds for linear forms in logarithms, notably Matveev's theorem, combined with reduction techniques in the spirit of Baker and Davenport. These methods allow us to derive effective bounds on the variables and to analyze the structure of possible solutions.

References

[1] A. Dujella and A. Pethő, *A generalization of a theorem of Baker and Davenport*, The Quarterly Journal of Mathematics.

[2] E. M. Matveev, *An explicit lower bound for a homogeneous rational linear form in logarithms of algebraic numbers. II*, Izv. Ross. Akad. Nauk Ser. Mat.

[3] B. M. M. de Weger, *Algorithms for Diophantine equations*, CWI Tract, Vol. 65, Stichting Mathematisch Centrum, Centrum voor Wiskunde en Informatica, Amsterdam, 1989.

[4] A. Baker and H. Davenport, *The equations* $3x^2 - 2 = y^2$ *and* $8x^2 - 7 = z^2$, Q. J. Math., 20(1):129–137, 1969.

[5] F. L. Mahadi Ddamulira and M. Rakotomalala, *Fibonacci numbers which are products of two Pell numbers*, The Fibonacci Quarterly, 54(1):11–18, 2016.

Study of Ricci solitons on the Thurston geometries Nil^4 and F^4

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Abstract.

In this talk, we consider the left invariant Riemannian metrics g_1 and g_2 on Lie groups Nil^4 and F^4, respectively. Then we introduce the notion of Ricci soliton and see when it is expanding, shrinking, steady and non-gradient. We classify Ricci solitons on $(Nil^4; g_1)$, $(F^4; g_2)$ and show that all Ricci solitons are expanding and non-gradient in the both spaces.

References

[1] Boukhari, H. Okbani, H. Mohammed cherif, A., On Ricci Solitons and Harmonic Vector Fields in the Thurston Geometry F^4, arXiv preprint arXiv: 2603.08969, (2026).

[2] Calvaruso, G.; De Leo, B. Curvature properties of four-dimensional generalized symmetric spaces. J. Geom. 2008, 90, 3046.

[3] Li, Y.; Cherif, A.M.; Xie, Y. Characterization of Ricci solitons and harmonic vector fields on the Lie group Nil^4. Mathematics 2025, 13, 1155.

[4] Filipkiewicz, R. Four Dimensional Geometries. Ph.D. Thesis, University of Warwick, Coventry, UK, 1983.

p-Biharmonic curves in three dimensional manifolds

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Abstract.

In this work, we give the conditions for the p-biharmonicity of the curves in three dimensional Riemannian manifold of constant curvature, and curves in three dimensional Sasakian space form. We construct a new example of proper p-biharmonic curves in the standard sphere $\mathbb{S}^3$.

References

[1] P. Baird, S. Gudmundsson, p-Harmonic maps and minimal submanifolds, Math. Ann. 294, 611-624 (1992).

[2] D. E. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Birkh*ä*user, Boston (2002).

[3] B. Bojarski, T. Iwaniec, p-Harmonic Equation and Quasiregular Mappings, Partial Differential Equations (Warsaw, 1984), 25-38. Banach Center Publ., vol. 19. PWN, Warsaw (1987).

[4] A. Fardoun, On equivariant p-harmonic maps. Ann. Inst. Henri. Poincar 15, 25-72 (1998).

[5] A. Mohammed Cherif, On the p-harmonic and p-biharmonic maps, J. Geom. **109** (41) (2018).

A Ricci Soliton Characterization of Pure Radiation Metrics Conformal to a Vacuum Space-Time

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Abstract.

A *Ricci soliton* is a natural generalization of an Einstein metric, and is defined on a pseudo-Riemannian manifold (M, g) by

$$L_X g + \varrho = \lambda g,$$

where X is a smooth vector field on M, L_X denotes the Lie derivative in the direction of X, ϱ is the Ricci tensor and λ is a real number. A Ricci soliton is said to be *shrinking*, *steady* or *expanding* according to whether $\lambda > 0, \lambda = 0$ or $\lambda < 0$, respectively. Moreover, we say that a Ricci soliton (M, g) is a *gradient Ricci soliton* if it admits a vector field X satisfying $X = \text{grad } h$, for some potential function h.

Ricci solitons are self-similar solutions of Hamilton's Ricci flow [3], and they are important since they were applied by Grigori Perelman to solve the long standing Poincaré conjecture. In recent years, there has seen much interest and increased research activities in Ricci solitons. They are of interests to physicists as well and are called quasi-Einstein metrics in physics literature (see [2]). For more details and further results we may refer to [1] and references therein.

In the context general relativity, a space-time which is viewed as a four-dimensional Lorentzian manifold, is said to be due to a pure radiation field if its energymomentum tensor T is of the form $T = \phi l \otimes l$, where l is a null vector and ϕ is a positive scalar function. Such a tensor contains most relevant information about mass density, energy density, momentum density etc. In this context, the Einstein field equations, which can be written as a set of nonlinear partial differential equations, relate the space-time geometry to the distribution of mass–energy, momentum and stress, by determining the metric tensor of spacetime or a given arrangement of stress–energy–momentum in space-time (see [4], [5]).

Using an explicit description of these metrics, we study the Ricci soliton equation and reduce it to a system of nonlinear partial differential equations. We completely solve this system and obtain a necessary and sufficient condition for such metrics to be Ricci solitons, together with the explicit form of the associated vector fields.

As an application, we show that these metrics admit gradient Ricci soliton structures only under a highly restrictive condition on the defining function involved in those metrics.

References

[1] H. D. Cao, Recent progress on Ricci solitons, Recent Advances in Geometric Analysis, 11 (2009), 1–38

[2] D. Friedan, Nonlinear models in $2+\varepsilon$ dimensions, Ann. Physics 163 (1985), 318–419.

[3] R. S. Hamilton, Three manifolds with positive Ricci curvature, J. Diff. Geom. 17 (1982), 255–306.

[4] G. Ludwig and S. B. Edgar, "Conformally Ricci flat pure radiation metrics," Class. Quantum Grav. 14 (12), 3453–3473 (1997).

[5] A. A. Shaikh, H. Kundu, M. Ali and Z. Ahsan, "Curvature properties of a special type of pure radiation metrics," J. Geom. Phys. 136, 195–206 (2019).

Harmonic and bi-harmonic homomorphisms between three-dimensional Riemannian unimodular Lie groups

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Abstract.

Let $\varphi : (M^m, g) \longrightarrow (N^n, h)$ be a smooth map between two Riemannian manifolds. The map φ is called harmonic if it is a critical point of the energy of φ defined by $E(\varphi) = \dfrac{1}{2}\int_M |d(\varphi)|^2 v_g$. The corresponding Euler Lagrange equation for the energy is given by the vanishing of the tension field

$$\tau(\varphi) = \mathrm{trace}_{\mathrm{g}}(\nabla)^2 d(\varphi) = \nabla^{\varphi}_{e_i}\nabla^{\varphi}_{e_i} d(\varphi) - \nabla^{\varphi}_{\nabla^M_{e_i}e_i} d(\varphi) \tag{6}$$

it is called bi-harmonic if it is a critical point of the bi-energy of φ defined by $E_2(\varphi) = \dfrac{1}{2}\int_M |\tau(\varphi)|^2 v_g$. The corresponding Euler Lagrange equation for the bi-energy is given by the vanishing of the bi-tension field

$$\begin{aligned}\tau_2(\varphi) &= -\,\mathrm{trace}_{\mathrm{g}}(\nabla)^2\tau(\varphi) - \mathrm{trace}_{\mathrm{g}}\, R^N(\tau(\varphi), d\varphi)d\varphi \\ &= -\Big[\nabla^{\varphi}_{e_i}\nabla^{\varphi}_{e_i}\tau(\varphi) - \nabla^{\varphi}_{\nabla^M_{e_i}e_i}\tau(\varphi)\Big] - R^N(\tau(\varphi), d\varphi(e_i))d\varphi(e_i).\end{aligned} \tag{7}$$

where $(e_i)_{i=1,..,m}$ is a local frame of orthonormal vector fields, and R^N is the curvature of ∇^N given by

$$R^N(X,Y) = \nabla^N_X\nabla^N_Y - \nabla^N_Y\nabla^N_X - \nabla^N_{[X,Y]}$$

Let (G, g) be a Riemannian Lie group, i.e., a Lie group endowed with a left invariant Riemannian metric. If $\mathfrak{g} = T_eG$ is its Lie algebra and $<,>_{\mathfrak{g}}= g(e)$, then there exists a unique bilinear map $A : \mathfrak{g}\times\mathfrak{g} \longrightarrow \mathfrak{g}$ called the Levi-Civita product associated to $(\mathfrak{g}, <,>_{\mathfrak{g}})$ given by the formula:

$$2 < A_u v, w >_{\mathfrak{g}}=< [u,v]^{\mathfrak{g}}, w >_{\mathfrak{g}} + < [w,u]^{\mathfrak{g}}, v >_{\mathfrak{g}} + < [w,v]^{\mathfrak{g}}, u >_{\mathfrak{g}} . \tag{8}$$

the couple $(\mathfrak{g}, <,> \mathfrak{g})$ defines a vector denoted $U^{\mathfrak{g}}$ by

$$< U^{\mathfrak{g}}, v > \mathfrak{g} = \mathrm{trace}(\mathrm{ad}_v),\ \ for\ any\ v \in \mathfrak{g}.$$

for any orthonormal basis $(e_i)_{i=1}^m$ of $\mathfrak{g}$,

$$U^{\mathfrak{g}} = \sum_{i=1}^{m} A_{e_i} e_i.$$

Note that $\mathfrak{g}$ is unimodular if and only if $U^{\mathfrak{g}} = 0$.
The differential $\xi : \mathfrak{g} \longrightarrow \mathfrak{h}$ of φ at e is a Lie algebra homomorphism.
A section X of $T^{\varphi}H$ is called left invariant if,

$$for\ any\ a \in G,\ a.X = X.$$

For any left invariant section X of $T^{\varphi}H$, we have for any $a \in G$, $X(a) = (X(e))^{\ell}(\varphi(a))$. Thus the space of left invariant sections is isomorphic to the Lie algebra $\mathfrak{h}$. Since φ is a homomorphism of Lie groups, g and h are left invariant, one can see easily that
$\tau(\varphi)$ and $\tau_2(\varphi)$ are left invariant and hence φ is harmonic (resp.biharmonic) if and only if $\tau(\varphi)(e) = 0$ (resp.$\tau_2(\varphi)(e) = 0$).
Now, one can see easily that

$$\tau(\xi) := \tau(\varphi)(e) = U^{\xi} - \xi(U^{\mathfrak{g}}),$$

$$\tau_2(\xi) = -\sum_{i=1}^{n} \Big(B_{\xi(e_i)} B_{\xi(e_i)} \tau(\xi) + \mathrm{K}^H(\tau(\xi), \xi(e_i))\xi(e_i) \Big) + B_{\xi(U^g)} \tau(\xi),$$

where $U^{\xi} = \sum_{i=1}^{m} B_{\xi(e_i)} \xi(e_i)$, B is the Levi-Civita product associated to $(\mathfrak{h}, <,>_{\mathfrak{h}})$ and $(e_i)_{i=1}^m$ is an orthonormal basis of $\mathfrak{g}$.

References

[1] P.Baird and J.C. Wood, Harmonic morphisms between Riemannian manifolds. Oxford Science Publications, 2003.

[2] S. Boubekour and M. Boucetta, Harmonic and biharmonic homomorphisms between Riemannian three dimensional unimodular Lie groups. Journal of Geometry and Physics, Volume 164. June 2021.

[3] M. Boucetta and S. Ouakkas, Harmonic and biharmonic homomorphisms between Riemannian Lie groups. Journal of Geometry and Physics volume 116. June 2017, 64-80.

[4] K.Y. Ha and J.B. Lee Left invariant metrics and curvatures on simply connected three-dimensional Lie groups. Math.Nachar. 282 (2009), 868-898.

Harmonicity on contact pair manifolds

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Abstract.

Let (M, g) be a Riemannian manifold, (T_1M, g_s) the unit tangent sphere bundle on M equipped with the Sasaki metric and $V : M \to T_1M$. If M is compact and orientable, the energy E(V)of V is defined by

$$E(V) = \frac{1}{2}\int ||dv||^2 v_g,$$

where v_g denotes the volume element of (M, g). V is called a harmonic vector field if it is critical for the energy functional E defined on the set $\Gamma(T_1M)$. V is a harmonic vector field if "$\overline{\Delta}V$ is collinear to V".
A contact pair manifold is defined firstly by D. Blair [6] under the name *bicontact* manifold. This structure has been developed by A. Hadjer and G. Bande at same manner as the contact structure [1, 2, 3, 4, 5]. They give some special cases on which the structure has a special forme of covariant derivative, Riemannian curvature and the (1,1)-tensor field h. Among these cases we have contact pair manifold with φ decomposable and the normal contact pair manifold.
Our objective was to study the harmonicity on contact pair manifolds.
A contact pair (α_1, α_2) of 1-forms on manifold $M^{2m+2n+2}$ where m, n are positive integers, is said to be a contact pair of type (m, n) if

$$\alpha_1 \wedge \alpha_2 \wedge (d\alpha_1)^m \wedge (d\alpha_2)^n \neq 0, \quad (d\alpha_1)^{m+1} = 0, \quad (d\alpha_2)^{n+1} = 0.$$

The Reeb vector fields Z_1, Z_2 are determined by

$$\alpha_h(Z_k) = \delta_h^k, \qquad i_{Z_h} d\alpha_k = 0, \qquad h, k = 1, 2,$$

where i_X is the contraction with the vector field X.
A contact pair structure on a manifold M is a triple $(\alpha_1, \alpha_2, \varphi)$, where (α_1, α_2) is a contact pair and φ a (1,1)-tensor field such that:

$$\varphi^2 = -Id + \sum_{k=1}^{2} \alpha_k \otimes Z_k, \qquad \varphi Z_k = 0, \quad k = 1, 2.$$

The characteristic foliations $\mathcal{F}_1, \mathcal{F}_2$ of M are determined by the integrable subbundles $\{X/\alpha_k(X) = 0, i_X d\alpha_k = 0\}$, $k = 1, 2$, respectively, then the tangent bundle of M is composed as follows

$$TM = T\mathcal{F}_1 \oplus T\mathcal{F}_2.$$

The endomorphism φ is said to be *decomposable* if $\varphi(T\mathcal{F}_i) \subset T\mathcal{F}_i$, for $i = 1, 2$.
We discuss the harmonicity of the unit vector field $\widetilde{Z} = \frac{1}{2}Z$. The results obtained :

Theorem 10.6. *Let $(M^{2m+2n+2}, \alpha_1, \alpha_2, \varphi, g)$ be a metric contact pair manifold with decomposable φ. If Z is a Killing vector field then $\widetilde{Z}$ is harmonic if and only if $m = n$.*

Corollary 10.7. *Let $(M^{2m+2n+2}, \alpha_1, \alpha_2, \varphi, g)$ be a normal metric contact pair manifold with decomposable φ. Then $\widetilde{Z}$ is a harmonic vector field if and only if $m = n$.*

Theorem 10.8. *Let $(M^{2m+2n+2}, \alpha_1, \alpha_2, \varphi, g)$ be a metric contact pair manifold with decomposable φ. Then*

$$\overline{\Delta} Z = 4(mZ_1 + nZ_2) - QZ. \tag{9}$$

Theorem 10.9. *Let $(M^{2m+2n+2}, \alpha_1, \alpha_2, \varphi, g)$ be a metric contact pair manifold with decomposable φ. Then $\widetilde{Z}$ is a harmonic vector field if and only if $pr_{|\mathcal{H}} QZ = 0$ and $4m - \alpha_1(QZ) = 4n - \alpha_2(QZ)$.*

Corollary 10.10. *Let $(M^{2m+2n+2}, \alpha_1, \alpha_2, \varphi, g)$ be a metric contact pair manifold $(\alpha_1, \alpha_2, \varphi, g)$ with decomposable φ. Then $\widetilde{Z}$ is a harmonic vector field if $m = n$ and Z is an eigenvector of Ricci operator.*

References

[1] G. Bande, Formes de contact généralisé, couples de contact et couples contacto-symplectiques, Thèse de Doctorat, Université de Haute Alsace, Mulhouse, 2000.

[2] G. Bande and A. Hadjar, Contact pairs, Tôhoku Math. J. 57 (2005),247-260.

[3] G. Bande and A. Hadjar, On normal contact pairs, Internat. J. Math. 21 (2010), 737-754.

[4] G. Bande and A. Hadjar, Contact pairs structures and associated metrics, Differential Geometry - Proceedings of the 8th International Colloquium, World Sci. Publ. (2009), 266-275.

[5] G. Bande and A. Hadjar, On the characteristic foliations of metric contact pairs, Harmonic maps and differential geometry. Contemp. Math. 542, Amer. Math. Soc., Providence, RI, pp. 255-259 (2011).

[6] D. E. Blair, G. D. Ludden and K. Yano, Geometry of complex manifolds similar to the Calabi-Eckmann manifolds, J. Differential Geometry 9 (1974), 263-274.

[7] G. Wiegmink, Total bending of vector fields on Riemannian manifolds, Math. Ann. 303 (1995) 325344.

Laplacian Equation with Kirchhoff Term on Riemannian Manifold

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Abstract.

Using variational methods and critical points theory, we obtained a smooth solution of class C^2, and it is unique when we have a singular term.

References

[1] Mohamed El Farouk Ounane, Kamel Tahri, *Existence and uniqueness results for a singular Kirchhoff type equation on a closed manifold*, Differential Geometry and its Applications, Volume 93, 2024.

Some Generalization of Zeckendorf's Lucas code

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Abstract.

Zeckendorf's Theorem states that every positive integer N can be uniquely decomposed as a sum of non consecutive Fibonacci numbers F_n, where $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. There are generalizations of Zeckendorf's Theorem for every monotone sequence a_n of distinct natural numbers for which $a_1 = 1$ and $a_n = a_{n-1} + a_{n-2}$. This paper aims to define Zeckendorf's code for certain generalizations of Lucas numbers. We first demonstrate that every positive integer N can be decomposed as a sum of Tribonacci-Lucas numbers H_n defined by the recurrence $H_n = H_{n-1} + H_{n-2} + H_{n-3}$ with initial terms $H_0 = 2$, $H_1 = 1$, $H_2 = 3$. We then propose a modified version of related sequence $(K_n)_{n\geq 0}$ defined by the same recurrence but with different initial terms $K_0 = 3$, $K_1 = 1$, $K_2 = 3$ to also represent every positive integer as a sum of its terms.

References

[1] S. Abbad, H. Belbachir, and B. Benzaghou, Companion sequences associated to the r-Fibonacci sequence: algebraic and combinatorial properties, Turkish J. Math **43** (2019), 1095–1114.

[2] S. Abbad, H. Belbachir and R. Ould-Mohamed, Lucas cube vs Zeckendorf's Lucas code, Montes Taurus J. Pure Appl. Math. **3**(2) (2021), 47–50.

[3] H. Belbachir and F. Bencherif, Linear recurrent sequences and powers of a square matrix, Integers **6** (2006), #A12, 17 pp.

[4] J. L. Brown, Jr., A new characterisation of the Fibonacci numbers, The Fibonacci Quarterly **7** (1965), 1–8.

[5] J. L. Brown, Jr., Unique representations of integers as sums of distinct Lucas numbers, The Fibonacci Quarterly **7** (1969), 243–252.

[6] P. S. Bruckman, The generalized Zeckendorf Theorems, The Fibonacci Quarterly **27** (1989), 338–347.

[7] M. Catalani, Identities for Tribonacci-related sequences, Cornell University Library (2002), available at https://arxiv.org/pdf/math/0209179.pdf.

[8] W. J. Hsu, Fibonacci cubes a new interconnection topology, IEEE trans. Parallel Distrib. Syst. **4** (1993), 3–12.

[9] T. J. Keller, Generalizations of Zeckendorf's Theorem, The Fibonacci Quarterly **1** (1972), 95–102.

[10] N. J. A. Sloane, The encyclopedia of integer sequences.

[11] E. Zeckendorf, Representation des nombres naturels par une somme de nombres de Fibonacci ou de nombres de Lucas, Bulletin de la société Royale de Sciences de Liège **41** (1972), 179–182.

Analysis and Solvability of Abstract Partial Differential-Algebraic Equations

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Abstract.

This work addresses the study of abstract evolution equations in Hilbert spaces X characterized by a singular leading operator M, situating the problem within the class of Partial Differential-Algebraic Equations (PDAEs). The system is defined by:

$$M\frac{dU(t)}{dt} = AU(t), \qquad t \geq 0,$$

where M is a singular diagonal matrix of rank $r < n$. The singularity of this operator precludes direct inversion and renders the classical theory of C_0-semigroups inapplicable in its original form.

To overcome this difficulty, this work presents a structural decomposition method of the state space into a Cartesian product $X = X_1 \times X_2$. This approach allows for the projection of the system onto its differential and algebraic components, leading to a block matrix representation of the linear operator A:

$$A = \begin{pmatrix} A & B \\ K & -L \end{pmatrix}.$$

We demonstrate how this semi-explicit reformulation facilitates the analysis, specifically by treating algebraic constraints as bounded couplings relative to the principal unbounded operator $\mathcal{A}$.

The primary objective of this study is to establish the well-posedness of the system. By imposing regularity conditions on the coupling operators, we prove the existence and uniqueness of the solution within a rigorous functional framework. This approach provides a unified analytical framework for solving singular evolution systems and offers new perspectives for studying complex evolution problems in mathematical physics.

Keywords: Partial Differential-Algebraic Equations (PDAEs), C_0-semigroups, Well-posedness, Stability estimates.

References

[1] S. A. Benabdallah and M. Souilah, "Global Existence and Uniqueness of Solutions to PDAEs with Applications," *Siberian Mathematical Journal*, vol. 67, no. 1, pp. 45–58, 2026. DOI: 10.1134/S0037446626010143.

[2] B. Z. Guo and L. Zhang, "Local null controllability for a chemotaxis system of parabolic-elliptic type," *Systems Control Letters*, vol. 65, pp. 106–111, 2014.

[3] K. A. Morris and A. O. Ozer, "Modeling and stabilizability of voltage-actuated piezo-electric beams with magnetic effects," *SIAM Journal on Control and Optimization*, vol. 52, no. 4, pp. 2371–2398, 2014.

[4] B. Jacob and K. Morris, "On solvability of dissipative partial differential-algebraic equations," 2022.

[5] Z. Ge, X. Ge, and J. Zhang, "Approximate controllability and approximate observability of singular distributed parameter systems," *IEEE Transactions on Automatic Control*, vol. 65, no. 5, pp. 2294–2299, 2019.